Dynamic survey measurement error in annual earnings

The role of cognition, earnings shocks and untaxed earnings*

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PRELIMINARY AND INCOMPLETE. PLEASE DO NOT CITE.[1]

Abstract

I compare survey reports of annual earnings in a large and long panel dataset, the Health and Retirement Study, to administrative data from the Master Earnings File of the Social Security Administration. Validation studies in the literature based on cross sectional data identified strong violations of the classical measurement error model. Most importantly the error, defined as the difference between the survey reports and the administrative records, is found to correlate negatively with earnings, meaning that high income people tend to underreport while low income people tend to overreport their earnings on average. The literature provided several mechanisms that can rationalize this finding, but cross sectional data could not differentiate between these models. The mechanism, however, is important, because these models have different implications for biases in applied work. My panel data, however, enables me to test these alternative models. I find evidence for two of these mechanisms. First, transitory earnings shocks are severely underreported in surveys, and the extent of underreporting is related to the cognitive capacity of the respondents. Second, I find evidence for measurement error in the administrative data, too. It is identified from separating earnings into permanent and transitory components in both the survey and the administrative data and test if permanent "measurement error" is related to food consumption after controlling for permanent earnings, income, wealth and other variables. I also find evidence that the mean-reverting property of measurement error is almost entirely driven by individuals with low earnings in the administrative data. Inspired by the psychology literature on survey response I propose three complementary economic models that can rationalize these finding. Finally I lay out a structural estimation model that encompasses all of theses mechanisms. I discuss identification and various problems. Future work will carry out the structural analysis.

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1 Introduction

Even though it is long known that survey data is prone to measurement error, there is a long tradition in applied economics to use survey data as if it were exactly measured. With better models of survey response error, researchers should be able to use these data in ways that take into account potential biases in applied work. Validation studies that compare survey reports to highly accurate records are very helpful to achieve this goal. Acquiring validation data, however, is not easy. First, there might not exist high accuracy data that the survey response could be compared to. Second, these records are sensitive for confidentiality reasons, therefore a lot of effort has to be made to convince data owners to provide them to researchers and allow it to be merged with survey data. Despite these obstacles numerous validation studies have been conducted in the last decades. In economics one of the most studied variables is earnings, and the accurate records either come directly from employers or from administrative sources like the IRS.

The first validation studies of earnings focused on the mean of the error and the correlation between the administrative and survey reports, but in the 80s and 90s the focus gradually shifted toward testing the assumptions of the classical measurement error model and estimating the potential bias in using the error ridden survey reports in regressions. The handbook chapter John Bound, Charles Brown and Nancy Mathiowetz (2001) provides a detailed overview. There is a couple of robust findings in these papers. The first is that the mean error is small and usually insignificant. The second finding is that measurement error is not classical because the correlation between the error and the true earnings is negative.[2] This is known as the "mean reverting property of the error", as the negative correlation implies that low income people tend to report too high and high income people tend to report too low earnings in surveys. Using a non-parametric estimation strategy Bollinger (1998) found that mean-reversion is mostly driven by the lowest earner individuals and at medium and high earnings mean-reversion basically disappears. Kristensen and Westergaard-Nielsen (2006) found the same result using Danish data.

The mechanism that leads to the mean-reverting property or survey reports, however, is not well understood, because the cross-sectional dataset that have been used in the literature cannot distinguish between alternative models. Pischke (1995) hypothesized that people underreport their transitory

[2] The classical measurement error model assumes that the error is independent of the true value.

income shocks which causes mean-reversion as people with positive/negative shocks report less positive/negative values than the truth. Kapteyn and Ypma (2007) point out that mismatch of individuals in the administrative and survey data can also cause mean-reversion, because, on average, each mismatched individual is matched to an average earner. Similarly, it is also possible that mismatched years between the datasets cause mean-reversion if there is a tendency in earnings to regress to an individual specific mean. Abowd and Stinson (2013) argue that measurement error in the administrative data can also lead to mean-reversion. One mechanism that can lead to measurement error in the administrative data is the existence of untaxed jobs.[3] They point out, though, that it is hard to distinguish between this mechanism and others, because the alternative models imply the same correlation structure in earnings. They use a partial identification framework to bound the measurement error in the administrative data, but they find quite uninformative bounds. I, however, will be able to provide more informative bounds by separating earnings into a permanent and transitory component and using consumptions data.

These models are able to predict mean-reversion, but as far as I know the literature has not yet discussed why the majority of mean-reversion is concentrated at low earnings values. My paper proposes three models that are all based on the ones above. Perhaps the simplest model is that untaxed earnings are more likely among employees with little taxed earnings. This can happen if untaxed work crowds out taxed work. The second model, which I call the Selective Memory model, assumes that people are more likely to underreport negative than positive transitory shocks. In the psychology literature Wagenaar (1986), Thompson et al. (1996) and Tourangeau et al. (2000) discuss that people tend to remember emotionally more intensive events better, but pleasant events are better stored in the memory than unpleasant events. Moreover, psychologists observed that people, when face a survey question about sensitive topics, tend to bias their answers toward more favorable outcomes. Some people might think that earnings is a gauge of personal success and they might find it embarrassing to report a too low earnings level. It is possible, for example, that people are more likely to report their usual earnings when they received a big negative transitory income shock but they report their total earnings after a positive shock. The third model I propose, the Recall Error model, assumes that transitory shocks are underreported in surveys, but that the extent of underreporting is stronger for people with low cognitive skills who are at the low end of the earnings distribution on average. I shall show that this recall error model produces a non-monotonic relationship between measurement error

[3]XXX Add citations to relevant public policy papers!!!

and true earnings, but the overall picture is similar to the empirical one. The Recall Error model is also related to the relevant psychology literature. Tourangeau et al. (2000) claim that there are four more or less distinct phases of the survey response: comprehension; retrieval; judgment and response. Comprehension is about understanding the survey questions, retrieval is about pulling all the relevant information together from the memory, judgment is about making manipulations on the information if necessary and response is about selecting one answer and communicating it. Error can occur at any of these phases, and it can correlate with the cognitive capacities of individuals. In the comprehension phase one might misunderstand the question or skip some important instructions.[4] When asked about earnings, people might mistakenly understand net instead of gross wages, or total income rather than wages and salary, etc. In the retrieval phase people can fail to recall some crucial information from the memory. In relation to earnings, people might forget about the exact amount of bonuses, tips, or the compensation for overtime work especially if they received them long ago. In the judgment phase people might make a math error when adding up the different earnings components, and in the response phase some might choose to answer a rounded number instead of a precise amount.

After discussing these models I test them using the Health and Retirement Study (HRS) merged to the Master Earnings File of the Social Security Administration. The main disadvantage of HRS is that its sample is the 50+ population. I will show that in the 50-65 subsample I use in this project, however, the properties of measurement error are very similar to other validation studies. Moreover, I argue that HRS has a couple of features that makes it very useful to learn about measurement error. The first is that it is a relatively large and long panel dataset, that enables me to analyze a longer horizon dynamics in earnings and error. The second is that it has a very good section on the cognitive abilities of the interviewees, as well as measures of consumption.

I find that mismatch is unlikely to be an important issue in HRS. Transitory earnings shocks, however, are severely underreported, and it is related to cognition. For example, I find that the standard deviation of the error and mean reversion strongly increases with the within person standard deviation of log earnings and they increase more strongly among individuals with low cognitive capacities. I also find direct evidence that there is measurement error in the administrative data, by regressing food consumption on the log survey report of earnings and flexibly controlling for permanent and transitory earnings in the administrative records as well as other control variables such as household

[4]Survey methodologists and psychologists have long recognized this problem. A popular method trying to minimize the role of misunderstanding is to conduct cognitive interviews to test how interviewees interpret the questions. For a recent example see de Bruin et al. (2010) on how some people misinterpret questions about inflation.

income, wealth, demographics, etc. It is important to note that this exercise can only be carried out by long panel datasets that enables researchers to separate permanent and transitory shocks in earnings. In cross sectional data, the survey report can correlate with food consumption even after controlling for administrative earnings even if there is no measurement error in the records. This is the case if consumption is based on permanent earnings, not total earnings, and if people underreport transitory earnings shocks in surveys.

I also propose a structural model and estimation strategy that can precisely estimate the contribution of each sources of measurement error. I discuss identification, and ongoing problems with the estimation. Future research will hopefully resolve these problems and carry out the estimation.

The paper is organized as follows. Section 2 describes the data and provides simple descriptive properties of measurement error. Section 3 discusses and tests models that can rationalize the mean-reverting property. Section 4 proposes three models that can explain why only the low earnings values are mean-reverted. Section 5 proposes a structural model of earnings and measurement error and section 6 concludes.

2 Data and simple descriptive results

2.1 The Health and Retirement Study

The survey data used for this project is from the Health and Retirement Study. HRS is a biannual panel dataset initiated in 1992. The initial sample represented the 51-62 year old US population[5], but in 1998 older cohorts were added to cover everyone 51 and older. The sample is refreshed every six years by the current six year cohort of persons aged 51-57. HRS has several advantages over other datasets to learn about measurement error. The first is that it is a moderately large panel dataset, and thus the dynamic properties of measurement error can be analyzed. The SIPP panels that were used in recent validation studies cover 2.5 to 4 years of data each. With HRS we can analyze longer work histories. The second advantage is the innovative questionnaire design. Item non-response for earnings and income questions is typically a serious issue in surveys. HRS developed a so-called

[5] People who were born in 1931-1941. See http://hrsonline.isr.umich.edu/sitedocs/surveydesign.pdf

unfolding bracket question sequence that is designed to reduce the effect of item non-response. People who answer "Don't know" to any of the earnings questions get a couple of follow-up questions where they have to indicate whether their earnings are smaller or higher than some pre-determined threshold values. Table 1 shows that with this method the non-response rate to the question about wage and salary income in HRS fell below 5 percent which is pretty impressive compared to other surveys. The third advantage of HRS is the detailed data is had on cognition, demographics, earnings histories and consumption.

For this project I use three proxies of cognition: 1. *Total recall*, a repeated 10-item word recall test measuring episodic memory; 2. *Mental status*, a test of basic cognitive capacity and attention, such as counting from 20 to 1; and 3. *Vocabulary*: a test of knowledge or crystallized intelligence. See Ofstedal et al. (2005) and McArdle et al. (2009) for further details about these measures. Sometimes I use the 27 point Langa-Weir scale, which is a summary score of various different items (see Crimmins et al. (2011) for details).

HRS asks about annual gross earnings in the last calendar year in the following five categories: 1. income from self employment; 2 wage and salary income; 3 other income from professional practice or trade; 4 other income from tips, bonuses, commissions; 5 other income e.g. from second jobs or military reserves. In this paper I concentrate on earnings from employment, so I disregard income from the first category. In principle the administrative data which covers earnings on the W-2 tax forms should match up with the sum of the components 2-5, so I added up these numbers. [XXX Discuss Section Q vs Section J XXX]

In HRS some questions are asked on the individual level and some questions are asked on the household level. Earnings are in the latter category, which means that only one member of the household is reporting about earnings, but she is reporting separately about her own and her spouse's earnings. This member of the household is called the financial respondent and the interviewees can decide who is supposed to do this task. The financial respondent status is determined in each waves of the study separately, the status can change over time. Note that when the cognition variables are used in regressions, one has to assign the cognition score of the financial respondent to an earnings report and not the own cognition score.

2.2 The administrative data

The source of the administrative data is the Detailed Earnings Records (DER) derived from the Master Earnings File (MEF) of the Social Security Administration that is linked to HRS. For details about the MEF and the linking procedure see Olsen and Hudson (2009) and the documents on the HRS website.[6] The DER data is derived from the W-2 forms filed by employers to the Internal Revenue Service each year. It contains five earnings variables:

1. Total compensation: This variable amounts to the sum of the Box 1 values on each W-2 forms submitted on behalf of a person by all his employers. Total compensation includes wages, bonuses, non-cash payments and tips[7]. Total compensation typically does not include deferred payments such as contributions to a 401k plans, but certain plans are included.

2. Social security earnings: This variables is derived from the Box 3 values of the corresponding W-2 forms. There are two major differences between this variable and total compensation. The first difference is that social security earnings contain information on deferred compensation as well. The second difference is that this variable is capped at the taxable maximum, and thus high income values are not observed perfectly. The taxable maximum was changing year by year. In 2002 it was $80,400, for example, meaning that any earnings beyond this amount were missing.

3. Medicare earnings: This variable is based on the Box 5 values of the W-2 forms. Medicare earnings are almost identical to social security earnings. The main difference is between the taxable maximums used for the two measures. Before 1991 the medicare and the social security caps were identical. Since 1994 there is no limit on the taxable earnings for medicare, and between 1991 and 1993 the difference between the medicare and the social security taxable maximums were diverging.

4. FICA taxable self employment earnings: This variable is based on Form 1040 Schedule SE reported by the self employed to IRS. The variable is capped at the same amounts as the social security earnings.

[6]There are two relatively detailed documents under the data section at hrsonline.isr.umich.edu that can be accessed after free registration. Note that the social security data is not public, and thus only these documents are available but not the data. The website also provides detailed information about how to get permission to use the restricted data.

[7]Only tips that the employee reported to the employer. Allocated tips are not part of Box 1.

5. Medicare taxable self employment earnings: This variable is almost identical to the previous, but here the less restrictive medicare caps are used.

In this paper I do not use self-employment earnings as there is evidence that the administrative data is of poor quality for this earnings category. The correlations between the first three measures of earnings are generally high, but they are not identical. In principle the best quality data is the post 1994 values of the medicare earnings which is uncapped and it also contains information on deferred compensation. In this paper I use both the first and the third measure, which are based on the Box 1 and 5 values, but I do not use the second measure.

For confidentiality reasons HRS top-coded all the earnings variables. For people whose earnings were above $250,000 in a given year, we only have interval information, where the intervals are $250,000-$299,999; $300,000-$499,999 and $500,000 and above. HRS also rounded earnings below $250,000 to the closest multiple of $100, with the exception of $0-$49, where we can differentiate between a true $0 and a $1-$49 value.

2.3 Linking the HRS data to the administrative data

The main caveat of the linking procedure is that HRS needed to acquire written consent from each sample members in order to get the administrative information on them. HRS made a lot of effort to increase the participation rate, but it remained below 100 percent. Generally HRS has a relatively good coverage rate for earnings before 1992 (slightly above 80 percent) and moderately good coverage rate for earnings afterwards (around 60 percent). The 2006 and 2008 rates are very low so I decided against using these waves.[8] Table 2 shows the total number of year-observation units in HRS and the units with administrative earnings information. As we can see in the majority of the samples the attrition rate is quite big, a little over 30 percent. Sample 5 which uses only the 1992 wave of HRS has the smallest attrition with 13 percent. The sample I consider the most important is sample 3, which consists of people with age 50-65 and with positive HRS wage report and no self-employment income. The reason for the age restriction is that HRS only represents the older population and above

[8]The reason for the gradual drop in the consent rate is the following. Before 2006 a consent covered the current and all previous waves but not the future ones. Therefore consent was asked more times for earlier waves than later waves. Since 2006 it covers both past and future waves, so it is expected that the consent rate will both increase and flatten out in the long run.

65 the number of workers drops significantly. The reasons for restricting our attention to people with positive HRS wage reports are that 1. I will use log specification and thus people with zero wage will be dropped anyway, 2. if we compare sample 1 and sample 2 we can see that people with zero wage reports were less likely to give permission to HRS to collect administrative information on them. The reason for the self-employment restriction is to minimize the effect of misspecification of earnings for the self-employed.[9] The other samples break down sample 3 into categories where the measurement error is expected to be more/less severe. For example, I expected that the measurement error is smaller for full time workers, salary workers, financial respondents who report on their own earnings, and those who gave continuous earnings reports.

Table 3 shows the results of regressions where the dependent variable is a dummy for having DER earnings information. As long as there is no selection into giving consent all the regressors should have coefficients close to zero. As we can see this is not the case. Even tough the majority of the variables have insignificant effect on providing consent, not all of them do and the R-squared is also around the non-negligible 0.05. The effect of income is insignificant, but the point estimates predict a turned U-shape. Among the standard demographic variables only age and a dummy of black Americans is significant. According to these models older Americans are more likely (perhaps because they were asked more times), and African Americans are less likely to give permission to HRS. Even more interesting is the effect of the cognitive variables. People with better memory scores participate less and people with good vocabulary scores participate more. These effects are also robust to different control variables and they do hold as simple correlations as well (not shown in this paper). Why cognition is so important is not entirely clear. The vocabulary effect might indicate that people with better scores understand the consent forms better and they understand that they should not be worried about HRS misusing it.

2.4 Simple descriptive patterns

From now on Y_{it}^a and y_{it}^a denote the level and log of annual earnings in the administrative data; Y_{it}^s and y_{it}^s denote the level and log of annual earnings reported in the HRS, and the measurement error, m_{it} is defined as the difference between log reports and log records, $m_{it} = y_{it}^s - y_{it}^a$. Note that the term, measurement error, is not a consistent one, because if there is measurement error in both the

[9]As a sensitivity analysis I should try to use other definitions as well.

administrate and the HRS data, then m_{it} only captures the difference between the two. Nevertheless, to simplify the language, unless I make it clear, measurement error will refer to m_{it}.

The describe measurement error I provide two statistics. The first is the standard deviation of the measurement error, σ_m in various sub-groups. The second is the coefficient from the following regression:

$$m_{it} = \gamma_{m,y^*} y^*_{it} + v_{it} \tag{1}$$

As Bound et al. (2001) discuss, γ_{m,y^*} is a measure of mean-reversion. In the classical measurement error model γ_{m,y^*} is assumed to be zero. In the literature, however, the coefficient is found to be roughly between -0.1 and -0.15.

First let me summarize the way I created the earnings variables. As I described above, interviewees in HRS could either give a continuous earnings value, or an interval response. In some cases the latter is only a half open interval, for example we only know that the value is bigger than $100,000. The administrative data is always in intervals, but for the majority of the values the intervals are very narrow ($100). In the descriptive section I do not model the interval responses so I had to figure out a way to summarize the intervals with numbers. In order to avoid adding measurement error to the data I took a very conservative approach. If the reported interval (or value) and the record interval overlapped, I defined measurement error as zero and I assigned the middle point of the overlapping interval to both earnings variables. If the two intervals did not overlap, I assigned the values to minimize the measurement error. The only case left is when I had two right half open intervals. In this case I assigned 110 percent of the maximum of the two left points of the intervals to both earnings variables. I followed this procedure with both the Box 1 and Box 5 values of the W-2 forms. The two variables gave very similar results so I decided to use only the Box 1 values. Given that most of the raw data values are either continuous reports or narrow intervals from the records, this procedure does not matter much for the qualitative results of the paper. After these transformations, I deflated the values with the CPI to 2000 dollars, and I took the logarithm of the two earnings measures.

Figure 1 and 2 in the appendix show the total and a restricted histogram of the measurement error. Similarly to previous studies I found non-normal error distribution: it seems to be skewed to the right, and it has more mass close to zero and at the extremes than what the normal distribution with similar variance would imply. The histograms based on the other samples are very similar.

One of the most robust findings of the literature is the mean-reverting property of the report, that is, the error and the true earnings have negative correlation. Figure 3-5, show that the dependence is not linear. Instead, the error seems to be unbiased in the middle range and it is strongly upward biased only for low earnings values, below the 20th percentile, and it is weakly downward biased only for very high earnings. This finding is in line with that of Bollinger (1998) and Kristensen and Westergaard-Nielsen (2006). Table 4 and 5 show the regression versions of these findings. As we can see, mean-reversion in the total sample is large ($\hat{\gamma}_{m,y^*} = -0.154$), but by dropping the lowest earnings values, mean-reversion basically disappears. The relationship does not seem to be linear even in the restricted samples, which indicates that the correlation measures does not capture the relationship sufficiently well.

Table 6 shows the number of missing values, the number of zero values and the number of non-zero values in the earnings reports and administrative records. As we can see the zero values do not match perfectly. The zero reports are especially problematic as more than 4000 person-wave units with zero reported earnings had positive earnings according to the tax forms. Note that later I will only concentrate on the data where both the records and the reports are positive as I use the log form.

Next I look at the properties of measurement error in different subgroups. Subgroup analysis has been done in many other surveys, too, but my panel data enables more detailed analysis. As far as I know this is the first study that splits up the sample by the within-individual standard deviation of earnings; and by the number of submitted W2 forms in a given year. Table 7 shows that in fact measurement error is larger and mean-reversion is stronger in groups that experience stronger earnings fluctuations. For example, among those who work for the same employer for at least two years, the standard deviation of measurement error (σ_m) is 0.37 while for those who work for less than two years this number is 0.68. Mean-reversion shows a similar pattern, with values of -0.07 and -0.2 respectively. Similar differences can be found between full and part time workers, salaried and hourly-paid workers. Mean-reversion is the lowest when we exclude observations that are lower than $\$10,000$ in 2000 dollars and among people whose within person standard deviation of earnings ($\sigma_{y_i^a}$) is below 0.25. The standard deviation of measurement error is lowest, but still substantial, among the full-time, salaried workers who work at the same employer for at least 2 years, and among people with low fluctuation of earnings ($\sigma_{y_i^a} \leq 0.25$). Table 7 suggests that people, whose earnings are fluctuating more, have hard time recalling their exact annual earnings. To further investigate this possibility I grouped people into 3 categories based on their average cognition scores between 1992 and 2004, and recomputed the same measures in the lowest and highest cognition groups. Table 8 supports the above argument as among the cognitively most able people measurement error and mean-reversion are smaller in almost all groups. Take a look at, for example, the last three rows that divides the sample based on the stability of the earnings histories. Among people with the most stable earnings histories there is no difference between the highest and lowest cognition groups. As earnings fluctuations increase, however, the role of cognition increases, too. Table 9-12 in the Appendix show result when the lowest earnings values are excluded from the estimation. The results are qualitatively the same.

3 Discussion of the mechanisms that can cause mean-reversion

3.1 Mismatch

Imagine that the administrative data is the true annual earnings of individuals and measurement error has the classical properties. As Kapteyn and Ypma (2007) discuss an error during the matching of the two datasets, that is, the mismatch of a fraction of individuals can explain mean reversion, because, on average, each mismatched individual is matched to an average earner. Similarly it is possible that people misinterpret the question and answer based on a different year; or during the matching procedure years are getting mixed up. As long as there is a tendency in earnings to regress to an individual specific mean, mismatched years can also lead to mean-reversion.

Random matching error can occur if the identifiers in the survey or the administrative dataset are corrupted or not precisely known. This mechanism can particularly be relevant when probabilistic matching methods are used. This mechanism can be less relevant in cases when we have high confidence in the validity of the identifiers in both datasets. It seems to be a reasonable assumption that earnings are unrelated to the propensity of having corrupted identifiers and thus this hypothesis predicts a linear relationship between measurement error and the W2 earnings. As we saw in Figure 3-5, however, mean-reversion mostly concentrates at low earnings and the relationship is flat zero after the 20th percentile of earnings. I conclude that the mismatch hypothesis is unlikely to explain the mean reverting property in annual earnings in the HRS, because it is unreasonable to assume that only the lowest earners are mismatched in the data.

It is similarly unlikely that mismatched years create the mean-reverting property in measurement error. Mismatch of years can create a mean-reversion within individuals' earnings histories, but when all individuals are pooled together, the effect should miniscule as the large majority of the variation is coming from different individuals.

Overall I find it implausible that mismatch caused the strong mean-reversion I found in the data.

3.2 Underreporting transitory earnings shocks

As I argued above, underreported transitory earnings shocks can explain mean-reversion in measurement error. In the descriptive section I also showed evidence that, indeed, measurement error seems to have higher variance and mean-reversions seems stronger in groups that experience higher fluctuations in their earnings; and the problem is more apparent for people with low cognition.

Bound et al. (1994) show simple derivations that the mean-reverting property, compared to the classical measurement error model, reduces the attenuation bias when earnings are on the right hand side, but introduces another type of attenuation bias when income appears on the left hand side. However, this is typically not found in the data, when earnings are on the left hand side, predictors of earnings seem to be unbiased. Bound et al. (1994) also realized this, when they claimed that the reason for not finding the attenuation bias is perhaps due to the fact that people underreport a part of their earnings that is a deviation from the earnings of their "*reference group [...] being a coworker mean, or the worker's own permanent earnings, or some combination of the two*" (pp. 354).[10] To put it another way, the fact that there is no attenuation bias when earnings appear on the left hand side also indicates that measurement error is mostly in transitory earnings. Table 13 illustrates this point using the HRS. The first column is a regression of earnings on education, age, gender, race, tenure, financial respondent status and three cognition variables. Besides financial respondent status all the variables are components of permanent income but they should not be correlated with transitory income. I control for financial respondent status because these are the people who report their own

[10] Consequently it would be very important that researchers who have access to validation data not only reported γ_{m,y^*}, as is standard in the literature, but also tried to estimate the bias from actual regressions.

wage. The second column is the same regression but with the log measurement error on the left hand side. The third regression is the same as the second but here I also control for earnings. As long as the relationship between the measurement error and the two components of earnings is the same, we would expect zero coefficients for the non-earning variables in the third column and coefficients with opposing sign in the first and second columns. The fact that the coefficients in the third column are non-zero and except for the race dummies they all have the same sign as in the earnings equation means that the correlation between measurement error and transitory income is more negative than the correlation between the error and permanent income. Note also that the three cognition variables all have strong positive and significant effect on permanent income. This will be an important assumption in one of my models in the next section.

Table 14 and 15 in the appendix show further direct tests. 14 splits up the sample into six quantiles based on within individual earnings fluctuations. The table shows that mean reversion is monotonically increasing with earnings fluctuations, and mean reversion is practically zero for the groups with the most stable earnings profiles. Table 15 decomposes earnings into permanent and transitory components and tests the hypothesis directly. I used the following strategy:

1. For $k \in \{2, ..., 7\}$ I restrict the sample to individuals with at least k valid HRS reports and corresponding W2 records.
2. Then I deflate earnings with the CPI to 2000 dollars.
3. I compute permanent earnings as the log of the average deflated earnings separately for the HRS records and the W2 records.
4. I subtract permanent earnings from the log of deflated HRS and W2 records to get estimates for transitory earnings shocks.

Table 15 shows that mean reversion in transitory earnings shocks are larger than mean reversion in permanent earnings. Mean reversion in transitory earnings is quite robustly estimated to be between -0.15 and -0.25 no matter how many years per individual are used and whether low earnings values are included or not. Mean reversion in permanent earnings substantially varies by the subsamples used for estimation. Mean-reversion decreases if low earnings values are excluded from the sample, or if only individuals with long labor histories are considered. Nevertheless, it seems probable that there is some mean-reversion in permanent earnings, too.

3.3 Untaxed jobs

The literature almost always takes the stand that the administrative or employer based earnings records are accurate and therefore we can directly observe the measurement error in validation studies by subtracting the records from the survey reports. There are many reasons to believe that this assumption is not entirely true. The first issue is unofficial income that is by definition missing from administrative records.[11] The second problem with administrative records is definitional: what exactly are earnings? It is not obvious, for example how to think of some components of income such as health insurance, job training programs, awards, fringe benefits, cafeteria, etc. Are they all parts of the income or they are partly other costs of production? Third, even if we have a strong opinion on which components of earnings we want to use, it is not necessarily true that the administrative data measures that.[12] The fourth problem with administrative data is that even if it exactly matches up with our preferred definition of earnings, the survey question can still be different from that. Given that the legal definition of earnings is very complex, it is basically impossible to list all the relevant definitions and instructions on the survey questionnaire. And the last problem with the administrative data is recording and other type of data managing errors. Bound et al. (1994) report that some of the outliers in the data seems to indicate problems with the administrative data rather than problems with the survey data. They had this impression after finding that for a few individuals the reported earnings did not change much while the administrative earnings had big jumps. Abowd and Stinson (2013) also report on serious difficulties in creating a reliable administrative dataset.

Let m_{it}^s and m_{it}^a denote measurement error in the survey and administrative data respectively, and let y_{it}^* denote the true, unobserved earnings. Then the covariance between m_{it} and y_{it}^a can be written as

$$Cov\left(m_{it}, y_{it}^a\right) = Cov\left(m_{it}^s - m_{it}^a, y_{it}^* + m_{it}^a\right) = Cov\left(m_{it}^s, y_{it}^*\right) + Cov\left(m_{it}^s, m_{it}^a\right) - \tag{2}$$

$$-Cov\left(m_{it}^a, y_{it}^*\right) - V\left(m_{it}^a\right) \tag{3}$$

[11] One might even hope that survey reports for this population are more accurate as surveys are collected anonymously. Unfortunately we know very little on how people with unofficial income respond to questions about their earnings in surveys. Hurst et al. (2010) finds strong evidence that the self-employed, who are expected to have the most unofficial income due to tax incentives, seriously underreport their income in surveys as well.

[12] The information that is collected on a tax form, for example, is precisely regulated by the law. The regulations are very complex and they are not solely based on economic considerations. For an example we can take a look at the instructions on a W2 form. On the list of what should be included in Box 1 we can find "*The cost of accident and health insurance premiums for 2%-or-more shareholder-employees paid by an S corporation*"; "*Taxable cost of group-term life insurance in excess of $50,000*", "*Unless excludable under Educational assistance programs (see page 5), payments for non-job-related education expenses or for payments under a nonaccountable plan*", etc. It is hard to argue that there are no arbitrary elements in these instructions.

Mean reversion in the data ($Cov\left(m_{it}, y^a\right) < 0$) can mean two things. Under the assumption that there is no measurement error in the administrative data ($m^a = 0$) we can conclude that there is true mean reversion ($Cov\left(m^s, y^*\right) < 0$). Note, however, that it is also possible that all the three covariances are zero, and $Cov\left(m_{it}, y^a\right) = -V\left(m^a\right)$.

Some researchers suggested that there might be measurement error in the administrative data, too, that is, $y^a_{it} \neq y^*_{it}$. Testing this hypothesis, however, is difficult. One idea could be to test if measurement error predicts economic behavior, for example, consumption. According to the permanent income hypothesis, however, this strategy can be flawed. Imagine, for example, that the W2 earnings are indeed the true earnings of individuals, but HRS respondents underreport their transitory earnings shocks. The permanent income hypothesis predicts that consumption depends on permanent income rather than total income. Therefore in a regression of consumption on W2 earnings and measurement error would yield of positive coefficients on both predictors even if W2 earnings precisely measure total earnings. This is the main reason, researchers could not test for the validity of administrative earnings so far.

The permanent income hypothesis gives us a guide how to test for this hypothesis when panel data is available. I use the following strategy:

1. Decompose HRS and W2 earnings into permanent and transitory components with the same methodology as in the previous section.

2. Decompose all other household income into permanent and transitory components, too.

3. Regress consumption on measurement error in permanent earnings, measurement error in transitory earnings, a flexible polynomial of permanent and transitory W2 earnings, permanent and transitory other household income, year dummies and potentially other variables.

HRS collected information about food consumption (at home and eating out) in Wave 1, 2 and Wave 5 onward. For the following regressions I use values from all waves, but of course Wave 3 and 4 are missing due to lack of consumption data. An important problem with these regressions is that while consumption is based on contemporaneous data, earnings refer to last year information. Nevertheless, given that permanent earnings is defined to be constant over time in real terms, it should not be a huge problem.

Tables 16 and 17 show the results when I restrict the sample to observations with at least 3 or 6 earnings values between 1992 and 2004 respectively. Tables 18-21 show results when the lowest earnings values are dropped from the sample. As expected, permanent earnings are stronger predictors of food consumption than transitory earnings, and the predicting power of earnings somewhat decreases with the inclusion of other covariates, such as other household income, wealth or demographics. More importantly, measurement error is a strong predictor of food consumption even after flexibly controlling for earnings, other household income, wealth and other demographics. This indicates measurement error in the W2 earnings values. The coefficients somewhat shrink but remain strong when low earnings values are dropped from the analysis. This indicates measurement error in the W2-s over the entire earnings distribution and not just at low earnings levels.

4 Discussion of why only low earnings values are mean reverted

In this section I briefly discuss three measurement models that are able to explain why the nonparametric regression of measurement error on the administrative earnings is a negative sloped convex function.

4.1 Untaxed Earnings model (UE)

Perhaps the simplest idea is that untaxed earnings crowd out taxed earnings, and thus we observe high earnings report in HRS at the lowest earnings values in the W2. Imagine the following model. There are 3 classes of people: 1. fraction p_1 of people whose entire earnings are taxed; 2. fraction p_2 of people whose entire earnings are untaxed; and 3. fraction p_3 of people who have some taxed and some untaxed earnings. Let's assume that average total earnings of individuals are the same in all three cases:

$$E\left(y_{it}^{1}\right)=E\left(y_{it}^{2}\right)=E\left(y_{it}^{3}\right)=E\left(y_{it}^{3,T}\right)+E\left(y_{it}^{3,U}\right),$$

where the superscript indicate the class of the individuals and whether the earnings type is taxed or untaxed. For precise derivation I also need distributional assumptions:

$$
\begin{aligned}
y_{it}^{1} &\sim N\left(\mu,\sigma^{2}\right)\\
y_{it}^{3,T} &\sim N\left(\mu^{3,T},\left(\sigma^{3,T}\right)^{2}\right)\\
Cov\left(y_{it}^{3,T},y_{it}^{3,U}\right) &= 0
\end{aligned}
$$

I also assume that $\sigma > \sigma^{3,T}$[13]. Finally let us assume that in HRS we observe the true earnings of individuals, but in the W2 data we only observe the taxed earnings. It immediately implies that measurement error has a positive mean:

$$E\left(m_{it}^{3}|y_{it}^{3}\right)=E\left(y_{it}^{3,U}|y_{it}^{3,T}\right)=E\left(y_{it}^{3,U}\right)>0$$

The expected value of measurement error condition on W2 earnings is

$$
\begin{aligned}
E\left(m_{it}|y_{it}^{a}\right) &= E\left(m_{it}|y_{it}^{a},g=1\right)\Pr\left(g=1|y_{it}\right)+E\left(m_{it}|y_{it}^{a},g=3\right)\Pr\left(g=3|y_{it}^{a}\right)\\
&= E\left(m_{it}|y_{it}^{a},g=3\right)\Pr\left(g=3|y_{it}^{a}\right)=\mu^{3,U}\Pr\left(g=3|y_{it}^{a}\right)
\end{aligned}
$$

The conditional probabilities can be computed from Bayes' theorem:

$$\Pr\left(g=3|y_{it}^{a}\right) = \frac{f\left(y_{it}^{a}|g=3\right)\Pr\left(g=3\right)}{f\left(y_{it}^{a}\right)}=\frac{\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}}{\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}+\frac{1}{\sigma}\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)\frac{p_1}{p_1+p_3}}$$

And thus

$$E\left(m_{it}|y_{it}^{a}\right)=\frac{\mu^{3,U}\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}}{\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}+\frac{1}{\sigma}\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)\frac{p_1}{p_1+p_3}}$$

Theorem 1. *Under the assumptions of the model $E\left(m_{it}|y_{it}^{a}\right)$ is a negative sloped convex function*

$$\frac{\partial E\left(m_{it}|y_{it}^{a}\right)}{\partial y_{it}^{a}} < 0 \tag{4}$$

$$\frac{\partial^{2} E\left(m_{it}|y_{it}^{a}\right)}{\left(\partial y_{it}^{a}\right)^{2}} > 0 \tag{5}$$

[13] Although a slightly weaker assumption is also enough.

Proof. See Appendix B. □

The intuition for the proof is simple: Very few people, who has untaxed earnings, have high y_{it}^a, and thus, when y_{it}^a is large, measurement error is close to zero. At low y_{it}^a the opposite is true, and thus, measurement error is expected to be large and positive there.

4.2 Selective Memory model (SM)

The decreasing and convex relationship between measurement error and true earnings can be explained by other models as well. The selective memory (SM) model I present in this section is based on the assumption that people are more likely to underreport negative than positive earnings shocks. Let us assume that true income is a sum of permanent and transitory income:

$$\begin{aligned} y^a &= y^p + y^t \\ \begin{pmatrix} y^p \\ y^t \end{pmatrix} &\sim N\left(\begin{pmatrix} \alpha \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_t^2 \end{pmatrix}\right) \end{aligned}$$

People report their permanent income precisely on average and they underreport a constant fraction of their transitory income depending on the sign of the transitory income:

$$y^s = \begin{cases} y^p + \tau_p y^t + \upsilon & \text{if} \quad y^t > 0 \\ y^p + \tau_n y^t + \upsilon & \text{if} \quad y^t \leq 0 \end{cases}$$

The measurement error is

$$m = \begin{cases} (\tau_p - 1)\, y^t + \upsilon & \text{if} \quad y^t > 0 \\ (\tau_n - 1)\, y^t + \upsilon & \text{if} \quad y^t \leq 0 \end{cases}$$

I assume that υ is uncorrelated with everything else in the model. The following theorems summarize the properties of the measurement error. It can be shown that the average measurement error is positive as long as $\tau_p > \tau_n$.

Theorem 2. *Under the assumptions of the model average measurement error is positive as long as $\tau_p > \tau_n$ and equals to*

$$\mathbb{E}(m) = \frac{\tau_p - \tau_n}{4} \sigma_t \phi(0) \approx 0.1 \sigma_t (\tau_p - \tau_n) \tag{6}$$

Proof. See Appendix B. □

This model predicts non-zero average error, just as the UE model did.

Theorem 3. *Under the assumptions of the model the conditional expectation of measurement error is*

$$\mathbb{E}\left(m|y^a\right) = \left(\tau_n - 1\right) \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} \left(y^a - \alpha\right) + \left(\tau_p - \tau_n\right) \left[A + B\left(y^a - \alpha\right)\right]$$

where

$$\begin{aligned} A &= \frac{\sigma_t \sigma_p}{\sqrt{\sigma_t^2 + \sigma_p^2}} \phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p \sqrt{\sigma_t^2 + \sigma_p^2}}\right) \\ B &= \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} \Phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p \sqrt{\sigma_t^2 + \sigma_p^2}}\right) \end{aligned}$$

Proof. See Appendix B. □

Theorem 4. *Under the assumptions of the model, the derivative of the conditional expectation is*

$$\frac{\partial}{\partial y^a} \mathbb{E}\left(m|y^a\right) = \left(\tau_n - 1\right) \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} + \left(\tau_p - \tau_n\right) \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} \Phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p \sqrt{\sigma_t^2 + \sigma_p^2}}\right)$$

Proof. See Appendix B. □

The derivative reaches its maximum at $y^a \to \infty$ where its value is

$$\begin{aligned} \max \frac{\partial}{\partial y^a} \mathbb{E}\left(m|y^a\right) &= \left(\tau_n - 1\right) \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} + \left(\tau_p - \tau_n\right) \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} \\ &= \left(\tau_p - 1\right) \frac{\sigma_t^2}{\sigma_t^2 + \sigma_p^2} < 0 \end{aligned}$$

Thus the function is always downward sloping and at high values of earnings the slope is determined by the τ_p.

Theorem 5. *Under the assumptions of the model, the second derivative of the conditional expectations is*

$$\frac{\partial^2}{\partial \left(y^a\right)^2} \mathbb{E}\left(m|y^a\right) = \left(\tau_p - \tau_n\right) \frac{\sigma_t^3}{\sigma_p \left(\sigma_t^2 + \sigma_p^2\right)^{3/2}} \phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p \sqrt{\sigma_t^2 + \sigma_p^2}}\right) > 0$$

Proof. It is straightforward after theorem (4) □

Therefore the function is negative sloped and convex. We can also see that the function becomes linear only if $\tau_p = \tau_n$, that is, the rate of forgetting does not depend on the sign of the transitory income shock. The curvature of the function is higher if there is big difference in remembering positive and negative shocks, or if transitory income has higher variance. The curvature is the highest at the mean earnings ($y^a = \alpha$).

4.3 The Recall Error model (RE)

The model assumes that people on average report their permanent income accurately, while they forget a fraction of their transitory income shocks. Fitness of memory will not only affect the rate of remembering the transitory shocks, but also the level of the permanent income. The negative dependence between income and error comes from the fact that people forget to report a portion of their transitory income shocks. In order to get a convex-like dependence, like in Figure 4, I need to incorporate a factor that influences the rate of forgetting and is correlated to permanent income.

The RE model is based on the assumption that transitory income is underreported, because people forget to report some components of income. However, it is also possible that underreporting is deliberate. The optimal prediction error model of model of Hyslop and Imbens (2001) assumes that people do not observe some variables precisely but they predict it given some signals.[14] It turns out that if we apply their model to reporting about transitory income, we get an observationally very similar model to the RE model. If people carry out the prediction optimally[15], they put a less than one weight on the otherwise unbiased signal, which leads to mean reversion, for example.[16]

[14]The behavior of the measurement error in these models depends heavily on the information set used to carry out the prediction, but in general it is very different from the classical measurement error model. Under some conditions, and as opposed to the CLM model for example, measurement error leads to no bias when the error ridden variable is on the right hand side of a regression, but leads to attenuation bias when it is an outcome variable. Hyslop and Imbens (2001) also show that standard instrumental variable techniques might mislead us. For example, under some conditions, when measurement error is on the right-hand side, OLS is unbiased, while IV is *biased away from zero*, which can easily mislead our interpretation of the difference between the estimates and make us draw a wrong conclusion that the OLS estimate is downward biased.

[15]See also Hoderlein and Winter (2009) who try to generalize this model to potentially less optimal cases.

[16]Let us assume that instead of observing transitory income y^t, people observe a corrupted version of it $\tilde{y}^t = y^t + \eta$ where $\eta \sim N\left(0, \sigma_\eta^2\right)$ and is independent of y^t. Interviewees also know the distribution of their transitory income, which is normal $y^t \sim N\left(0, \sigma_t^2\right)$. When receiving a signal $\tilde{y}^t$ they update their beliefs about their transitory income and they report $y^s = y^p + \mathbb{E}\left(y^t|\tilde{y}\right) = y^p + \tilde{y}^t \frac{\sigma_t^2}{\sigma_t^2+\sigma_\eta^2} = y^p + y^t \frac{\sigma_t^2}{\sigma_t^2+\sigma_\eta^2} + \eta \frac{\sigma_t^2}{\sigma_t^2+\sigma_\eta^2}$ where y^p is permanent income and y^s is reported income.

4.3.1 The setup of the model

Let us assume that true earnings can be decomposed into a permanent and a transitory component:

$$y_i^a = y_i^p + y_i^t$$

Transitory income might not be a simple white noise uncorrelated over time. I rather think of transitory income as unusual parts of components that are fluctuating year by year, for example unusual bonuses, tips, overtime work, income from secondary jobs, etc. These components can be correlated over time. Reported earnings will be:

$$y_i^s = y_i^p + \tau\left(c_i, z_i\right) y_i^t + v_i$$

where $\tau\left(c, z\right)$ is the rate of remembering the transitory income; c is cognitive capacity that will typically be negatively related to forgetting (and hence $\tau_c > 0$); z is other factors associated with forgetting but not with labor market performance, and v_i is a general error component not related to anything. You can think of z as a vector of observable variables like month of interview (whether it is a tax-filing month), and unobservable variables not related to earnings (such as using the tax forms when answering the survey or having a big transitory income shock right before the survey). Another important factor in z might be the unusualness of a given year in work history. We might expect that people who did not work, for example, at the beginning of the year but they generally do work might forget about the missing months in their work history. In the empirical part of the paper I will think of c as a vector, but as of now let us think of it as a single variable or an index created from several variables.

Generally you can think of $\tau\left(c_i, z_i\right)$ as a number between zero and one, but here I assume less:

$$\begin{aligned}
0 &\leq T\left(c\right) = \mathbb{E}_{z|c}\left[\tau\left(c, z\right)|c\right] \leq 1 \\
T'\left(c\right) &\geq 0, T''\left(c\right) \text{ exists} \\
\lim_{c\to-\infty} T'\left(c\right) &= \lim_{c\to\infty} T'\left(c\right) = \lim_{c\to-\infty} T''\left(c\right) = \lim_{c\to\infty} T''\left(c\right) = 0
\end{aligned}$$

$\tau\left(c_i, z_i\right)$ can be anything, all I require is that after integrating out everything except for cognition, the number should be between zero and one.[17] A crucial assumption of my model is that permanent

[17] The more general approach is motivated by psychology. It is well known that when people answer survey questions on issues from the past, they 1. tend to forget things that happened in the past; 2. they extrapolate from recent events. Let us say for example that someone who works as a waiter received unusually high amount of tips in the last two months

income depends on cognitive capacities:

$$y_i^p = \alpha_0 + \alpha_c c_i + \alpha_x x_i$$

where x_i is factors affecting earnings but not the rate of forgetting. I further assume that some of the random variables follow normal distribution:

$$\begin{pmatrix} c \\ x \\ y_i^t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & 0 & 0 \\ 0 & \sigma_x^2 & 0 \\ 0 & 0 & \sigma_t^2 \end{pmatrix}\right)$$
$$\implies \mathbb{V}\left(y^a\right) = \alpha_c^2\sigma_c^2 + \alpha_x^2\sigma_x^2 + \sigma_t^2$$

The zero mean assumption is just a normalization. I also assume that transitory income is uncorrelated with the components of permanent income and c and x are also uncorrelated. The last assumption can be justified if cognitive capacity has already been partialled out from x before included in the model.

The proposed model assumes that people remember their permanent income precisely, their transitory income partially and they also report with some error that is uncorrelated with everything.

$$m_i = y_i^s - y_i^a = \left[\tau\left(c_i, z_i\right) - 1\right] y_i^t + v_i \tag{7}$$

where m is measurement error.

4.3.2 Solution in simple cases

In this section I derive the expected value of measurement error conditional on true earnings and I check whether 1. it has negative slope everywhere; 2. whether it is linear, convex or ambiguous. For

or unusually low amount of tips a year ago. The psychological findings imply that this person will have a tendency to overreport his transitory income rather than underreporting it, meaning that for him $\tau\left(c_i, z_i\right)$ will exceed one. My model enables this to happen, all I require is that waiters underreport their transitory income on average. As it can be seen above I also require that cognition decreases the rate of forgetting, and I also provided some non-restrictive technical assumptions.

simplifying notation I will neglect the subscript i.

$$\begin{aligned}\mathbb{E}_{c,z,x|y^a}\left[m|y^a\right] &= \mathbb{E}_{c,z,x|y^a}\left[\left[\tau\left(c,z\right)-1\right]y^t+v|y^a\right]\\ &= \mathbb{E}_{c,z,x|y^a}\left[\left[\tau\left(c,z\right)-1\right]y^t|y^a\right]+\mathbb{E}_{c,z,x|y^a}\left[v|y^a\right]=*\end{aligned}$$

Note that the second term is zero, as the general error component is uncorrelated with earnings.

$$\begin{aligned}&= \mathbb{E}_{c,z,x|y^a}\left[\left[\tau\left(c,z\right)-1\right]y^t|y^a\right]+0 &(8)\\ &= \mathbb{E}_{c,z,x|y^a}\left[\left[\tau\left(c,z\right)-1\right]\left(y^a-y^p\right)|y^a\right] &(9)\\ &= -\mathbb{E}_{c,z,x|y^a}\left[\left[1-\tau\left(c,z\right)\right]\left(y^a-\alpha_0-\alpha_c c_i-\alpha_x x_i\right)|y^a\right] &(10)\end{aligned}$$

From here I proceed by checking the implications of different further assumptions:

1. The classical measurement error assumption is that $\tau\left(c,z\right)\equiv 1$ and thus there is only general independent error. In this case (10) simplifies to:

$$\mathbb{E}_{c,z,x|y^a}\left[m|y^a\right]=-\mathbb{E}_{c,z,x|y^a}\left[0\left(y^a-\alpha_0-\alpha_c c_i-\alpha_x x_i\right)|y^a\right]=0$$

   This result is rather tautological. In case people remember all of their transitory income shocks, the classical measurement error model implies that measurement error is uncorrelated with true earnings.

2. In case $\tau\left(c,z\right)=\tau\left(z\right)$ that is, forgetting does not depend on cognition:

$$\begin{aligned}\mathbb{E}_{c,z,x|y^a}\left[m|y^a\right] &= -\mathbb{E}_{c,z,x|y^a}\left[\left[1-\tau\left(z\right)\right]\left(y^a-\alpha_0-\alpha_c c_i-\alpha_x x_i\right)|y^a\right]\\ &= -\mathbb{E}_z\left[1-\tau\left(z\right)\right]\mathbb{E}_{c,x|y^a}\left[y^t|y^a\right]\\ &= -\left(1-\mathbb{E}_z\left[\tau\left(z\right)\right]\right)\mathbb{E}_{c,x|y^a}\left[y^t|y^a\right]=*\end{aligned}$$

   without distributional assumptions $\mathbb{E}_{c,x|y^a}\left[y^t|y^a\right]$ cannot be simplified any further. However, normality implies that

$$*=-\left(1-\mathbb{E}_z\left[\tau\left(z\right)\right]\right)\frac{\sigma_t^2}{\sigma_y^2}\left(y^a-\alpha_0\right) \qquad (11)$$

   The dependence between measurement error and earnings is negative and linear implying that the "covariance" between measurement error and income as a simple statistic captures the dependence between these variables sufficiently well. We can also see that the dependence is stronger if people forget more of their transitory income ($\mathbb{E}_z[\tau(z)]$ is small) and if the ratio of transitory income to total income is high. These are intuitive results.

3. In case $y^p = \alpha_0 + \alpha_x x_i$ (permanent income does not depend on cognition):

$$\mathbb{E}_{c,z,x|y^a}[m|y^a] = -\mathbb{E}_{c,z,x|y^a}[[1-\tau(c,z)](y^a - \alpha_0 - \alpha_x x_i)|y^a] \tag{12}$$

$$= -\mathbb{E}_{c,z}[[1-\tau(c,z)]]\,\mathbb{E}_{x|y^a}\left[y^t|y^a\right] \tag{13}$$

$$= -\left(1-\mathbb{E}_c[T(c)]\right)\frac{\sigma_t^2}{\sigma_y^2}(y^a-\alpha_0) \tag{14}$$

   the dependence is still negative and linear.

4. All is left is the general case, where both $\tau(c,z)$ and y^p truly depends on cognition. I devote the next section to this case as it is more complex.

4.3.3 Solution to the general case

In the general case $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ will not be linear and it may or may not be convex. In theorem 6, 7 and 8 I show the simplest form of the function, its derivative and its second derivative.

Theorem 6. *Under the assumptions of the model*

$$\mathbb{E}_{c,z,x|y^a}[m|y^a] = \frac{\sigma_t^2}{\sigma_t^2 + \alpha_x^2\sigma_x^2}\mathbb{E}_{c|y^a}[(1-T(c))(\alpha_0 + \alpha_c c - y^a)|y^a]$$

Proof. See Appendix B. □

As long as $T'(c) \neq 0$ this expression cannot be simplified any further. Generally the model implies that for low values of y^a it should be a positive number, while for high values of y^a it should be a negative one. Let us see the first derivative.

Theorem 7. *Under the assumptions of the model*

$$\frac{\partial}{\partial y^a}\mathbb{E}_{c,z,x|y^a}[m|y^a] = \frac{\sigma_t^2\alpha_c\sigma_c^2}{(\sigma_t^2+\alpha_x^2\sigma_x^2)\,\sigma_y^2}\mathbb{E}_{c|y^a}[(T'(c))(y^a-\alpha_0-\alpha_c c)|y^a] - \frac{\sigma_t^2}{\sigma_y^2}\mathbb{E}_{c|y^a}[(1-T(c))|y^a] \tag{15}$$

Proof. See Appendix B. □

As long as $T(c)$ is a constant, that is forgetting does not depend on cognition, the first term falls out, and the second term simplifies to the simple case characterized in equation (11). If $T(c)$ is not a constant, the first term can be positive for certain income levels, and it can even dominate the second term. The intuition is the following. The second term is the first order effect. It captures the fact that people forget a portion of their transitory income leading to a mean reverting property of survey income, and the mean reversion decreases with income as cognition is more likely to be high for high income people. The first term is a second order effect. An increase in y^a makes it more likely that someone is of high cognition with higher value of $T(c)$ that decreases the underreport of income. In case $T(c)$ is changing with cognition fast enough, the second order term can dominate the first order term.

Even if we cannot describe the behavior of the first derivative precisely, we can still say something about it. I assumed before that $\lim_{c\to-\infty} T'(c) = \lim_{c\to\infty} T'(c) = 0$. It implies that for low/high values of y^a the derivative will approximately be constant because the majority of the probability mass of $c|y^a$ will be for low/high values of cognition, where $(T'(c))$ is already approximately zero. Mathematically, if $T_l = \lim_{c\to-\infty} T'(c) < \lim_{c\to\infty} T'(c) = T_h$

$$\begin{aligned}\lim_{y^a\to-\infty} \frac{\partial}{\partial y^a}\mathbb{E}_{c,z,x|y^a}[m|y^a] &= -(1-T_l)\frac{\sigma_t^2}{\sigma_y^2}\\ \lim_{y^a\to\infty} \frac{\partial}{\partial y^a}\mathbb{E}_{c,z,x|y^a}[m|y^a] &= -(1-T_h)\frac{\sigma_t^2}{\sigma_y^2}\end{aligned}$$

The function $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ is approximately linear at the beginning and at the end, and it has higher slope at the beginning than at the end. In the middle range the function can be smooth (like in Figure 1 above) or it can have unexpected twists. The function will be smoother if the first term in (15) is small in magnitude, that is, if either $T'(c)$, α_c or σ_c^2 is small. The function can have strange properties in the middle if the forget function changes rapidly at some cognition levels and cognition takes up a large fraction in the variance of income. In reality we should not expect any of these to be true.

Now let us see the second derivative.

Theorem 8. *Under the assumptions of the model*

$$\frac{\partial^2}{\partial (y^a)^2}\mathbb{E}_{c,z,x|y^a}[m|y^a] = \frac{\sigma_t^2\alpha_c^2\sigma_c^4}{(\sigma_t^2+\alpha_x^2\sigma_x^2)\,\sigma_y^4}\mathbb{E}_{c|y^a}[[T''(c)(y^a-\alpha_0-\alpha_c c)]\,|y^a]$$

Proof. See Appendix B. □

The assumptions that $\lim_{c\to-\infty} T'(c) = \lim_{c\to\infty} T'(c) = 0$ but $T'(c) \neq 0$ imply that the second derivative is not necessarily always convex. In general we expect $T''(c)$ to be positive for smaller cognition levels and negative for high cognition levels (just like the normal c.d.f.) that would bring a positive correlation between $T''(c)$ and $(y^a - \alpha_0 - \alpha_c c)$ conditional on y^a but this is not always true.

4.3.4 Simulations

I show two types of simulations in this section. In the first part I use numerical integration using matlab to characterize the $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ function for different parameter values. Then I simulate data in stata, estimate $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ non-parametrically and compare it to Figure 1.

As I discussed above $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ is expected to be smoother if either $T'(c), \alpha_c$ or σ_c^2 is small, that is if cognition is not a strong predictor of both forgetting and earnings. Thus I will run sensitivity analysis based on these variables. I specify $\tau(c,z)$ as a probit:

$$\begin{aligned}\tau(c,z) &= \begin{cases}1 & \text{if} \quad \gamma_0+\gamma_1 c+z>0\\ 0 & \text{if} \quad \gamma_0+\gamma_1 c+z\le 0\end{cases}\\ z &\sim N(0,1)\end{aligned}$$

This implies

$$\begin{aligned}T(c) &= \mathbb{E}_{z|c}[\tau(c,z)\,|c] = \Pr(\gamma_0+\gamma_1 c+z>0) = \Pr(\gamma_0+\gamma_1 c>-z)\\ &= \Phi(\gamma_0+\gamma_1 c)\end{aligned}$$

γ_0 determines whether people with average cognition are more or less likely to remember their transitory income than 50 percent. A positive γ_0 indicates that people on average remember their transitory income. γ_1 determines the slope of $T(c)$. Higher γ_1 means higher $T'(c)$ and thus according to our expectations, more complex behavior of $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ in the middle range.

Numerical integration of $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ Here I do not intend to calibrate the model with real values, I only want to illustrate the function and its sensitivity. I make the following choices for default parameter values. For the variance terms I use higher variation in cognition than in transitory income, and I neglect x as it does not have a special role in the integral: $\sigma_c^2 = 2, \sigma_x^2 = 0, \sigma_t^2 = 1$. By default I use $\alpha_c = 1$ so that cognition has the same weight in earnings as transitory income. For the parameters of the $T(c)$ function I have chosen $\gamma_0 = 1.5$ and $\gamma_1 = 1$.

Let us see first the role of α_c which is the importance of cognition in earnings. Figure 5 shows that an increase in α_c leads to smaller bias for low income levels, which makes sense as the ratio of transitory income in total income decreases. The other observation is that $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ does not seem to be convex, especially not for high values of α_c. While the function seems to be linear at both ends, in the middle the second derivative of the function changes signs several times. Figure 6 shows the same graph for σ_c^2. This gives a very similar picture as the previous one. An increase in σ_c^2 decreases the bias but it also makes the function less smooth in terms of convexity. Lastly, Figure 7 shows the result for γ_1. The increase of this parameter makes the $T(c)$ function sharp at one point and relatively flat away from this point. As we can see on the graph this change corresponds to higher bias at the beginning, since for low values of y_a it is more likely that someone has low values of cognition and thus $T(c) = 0$. For high values of y_a $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ is already flat at zero. However, in the middle range there is a negative hump in the function. I do not see any important economic reasons for this negative hump. We can also see on the picture that if γ_1 is sufficiently low, the function has smaller bias at the beginning because $T(c)$ will be less likely to be close to zero. I also plotted $T(c)$ on Figure 8 for different choices of γ_1. As we can see, for high values of γ_1 $T(c)$ changes sharply in the relevant range, while for low values we only see a small portion of the normal c.d.f.

Simulation Here one goal of mine was to find parameters that make the output of my model similar to figure 1 at the previous section. I have chosen the following parameter values.

1. Variances: $\sigma_c^2 = \sigma_x^2 = 1.5, \sigma_t^2 = 1, \sigma_v^2 = 0.2$. Recall that σ_v^2 represents the portion of measurement error that is independent of everything in the model.

2. Coefficients in the earnings equation: $\alpha_0 = 10, \alpha_c = \alpha_x = 1$

3. Coefficients in $T(c)$: $\gamma_0 = 0, \gamma_1 = 0.4$

On the histogram of measurement error shown on Figure 9 we can see that the measurement error is not normally distributed as its kurtosis is high. This is because the distribution is a mix of two normal distributions: one with small variance (people who manage to report their transitory income) and one with high variance (people who forget about their transitory income). In the HRS data we can see a very similar measurement error distribution. The nonparametric regression of error on earnings on Figure 10 also gives similar results to the HRS data. $\mathbb{E}_{c,z,x|y^a}[m|y^a]$ starts off at a positive number, its slope decreases and it is flat at zero at the top. Moreover, it has many outliers at both ends, but the outliers at high income values tend to be negative, while the outliers at low income values tend to be positive. The correlation between measurement error and income is -0.3, and the regression of error on income brings a coefficient of -0.093.

5 Structural estimation

The goal of the structural estimation is to combine and estimate the three proposed models above in order to understand how important each mechanisms are. The model features the following

1. Taxed and untaxed earnings. Taxed earnings appear on the W2 forms, but untaxed earnings do not. Both taxed and untaxed earnings are composed of

   (a) A permanent component that follows a random walk.

   (b) A transitory component that follows an MA(1) process.

2. Measurement error in survey reports are due to:

   (a) Forgetting transitory earnings shocks. I assume that people forget to report transitory shocks with some probability. In a simple model people either report transitory shocks completely, or they forget it at all. In more general versions I might want to be more flexible on this assumption.

   (b) Negative transitory shocks might be more likely to be forgotten.

(c) The model also features a classical measurement error component.

3. Consumption

   (a) Consumption is assumed to depend on the sum of taxed and untaxed earnings in a flexible way. For identification I need that taxed and untaxed earnings affect consumption in the same way.

4. Rounding

   (a) Both W2 and survey reports can be rounded, and it is modeled in an interval regression fashion.

5. Missing information

   (a) This is a tricky part. First I will make the simplifying assumption that missing data is ignorable.

I plan on using a fully specified model and the estimation can be carried out by Markov Chain Monte Carlo.

5.1 Discussion of identification

The estimation of the dynamic permanent earnings model is quite standard in the literature. The identification of the untaxed earnings is coming from the food consumption equation. Individuals, who consume significantly more than what they are predicted based on the W2 earnings will be more likely to have untaxed earnings, especially if their HRS reports also exceed the W2 values. The permanent component of untaxed earnings might be easier to estimate. The transitory component can only be estimated if transitory earnings predict consumption. The rest of the parameters of survey response seem pretty straightforward to estimate. Forgetting is identified from the ratio of variances of transitory earnings in the W2-s and the HRS, etc.

5.2 Discussion of problems

So far I had trouble estimating even a simple dynamic earnings model using the W2 data only. The main problem seems to be that many individuals in the 50-65 age range are transitioning to retirement and consequentially there are large movements in and out of the labor force as well as between different jobs. My model seemed to work just fine when I restricted the sample to full year continuing spells, identified from tenure data in the HRS (see Table 22 in the appendix), but it did not work well when I tried to estimate the model on the full sample (Table 23). Right now I am working on cleaning the labor market histories in the HRS and I will try to estimate labor market transitions together with the dynamic earnings model. The main disadvantage of this approach is that the labor market history data likely contains a lot of measurement error, too.

6 Conclusion

This paper analyzed the properties of measurement error in a large panel dataset, the HRS. Consistently with the literature I found evidence for non-classical measurement error, namely negative correlation between the error and the true earnings. I also showed that this correlation is mainly driven by low earning values, at the middle and at the top earnings are well described by the classical measurement error model. Then I discussed and tested several mechanisms that can rationalize the mean-reverting property. I found that mismatch is unlikely to be an important issue in HRS. Transitory earnings shocks, however, are severely underreported, and the extent of underreporting is related to cognition. I also found direct evidence that there is measurement error in the administrative data, by regressing food consumption on the log survey report of earnings and flexibly controlling for permanent and transitory earnings in the administrative records as well as other control variables. I proposed three models that can also rationalize the fact that mean-reversion is concentrated at low earnings values. I proposed a structural model to estimate these three models jointly. Future research will carry out the estimation of this structural model.

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Table 1: Response pattern to wages/salary in HRS, 1992-2004

	N	Percent
1. continuous value	37,246	35.87
2. complete bracket	3,482	3.35
3. half open interval	907	0.87
5. no value given in bracket	2,188	2.11
6. no income	58,618	56.46
7. dk ownership	450	0.43
8. technical problem	206	0.2
9. no fin resp	729	0.7
Total	103,826	100

Table 2: Samples used, number of year-observation units and ratio with DER earnings

		Total number	With DER earnings	Percentage
Sample 1	Everyone 1992-2004	103826	57548	0.55
Sample 2	(1) + pos. wage report, no self-emp. inc.	39095	26105	0.67
Sample 3	(2) + 50-65 years old	31739	21308	0.67
Sample 4	(3) + DER earnings > $10,000 in 2000 $	16651	16651	1.00
Sample 5	(3) + 1992 sample	6400	5555	0.87
Sample 6	(3) + at least two years of tenure	22514	15196	0.67
Sample 7	(3) + less than two years of tenure	4976	3373	0.68
Sample 8	(3) + full time worker	21729	14728	0.68
Sample 9	(3) + not full time worker	10010	6580	0.66
Sample 10	(3) + salary worker	10269	7045	0.69
Sample 11	(3) + not salary worker	17485	11505	0.66
Sample 12	(3) + financial respondent	21424	14267	0.67
Sample 13	(3) + not financial respondent	10315	7041	0.68
Sample 14	(3) + DER outliers dropped*	31687	21256	0.67
Sample 15	(3) + gave continuous wage report	28605	19649	0.69

*See the text for the definition

Appendix A: Tables and Figures

Table 3: Linear regressions on the selectivity of sample 3

	Gave permission			
Log HRS wage	-0.002	-0.007	0.027	0.029
	[0.004]	[0.026]	[0.037]	[0.038]
Squared log HRS wage		0	-0.001	-0.001
		[0.001]	[0.002]	[0.002]
HRS wage zero	-0.106	-0.128	-0.023	0.018
	[0.093]	[0.145]	[0.199]	[0.204]
Education in yrs			0.004	0.001
			[0.002]	[0.002]
Age in yrs			0.004	0.004
			[0.001]**	[0.001]**
Female			0.017	0.024
			[0.014]	[0.015]
Hispanic			-0.03	-0.036
			[0.021]	[0.022]
Black			-0.093	-0.087
			[0.016]**	[0.017]**
Financial respondent			-0.019	-0.019
			[0.011]	[0.011]
Veteran			0.002	-0.002
			[0.015]	[0.015]
Tenure in yrs			0	0
			[0.000]	[0.000]
Log financial wealth			-0.001	-0.002
			[0.002]	[0.002]
Financial wealth zero			0.019	0.022
			[0.024]	[0.025]
Totall recall				-0.033
				[0.008]**
Mental status				0.009
				[0.009]
Vocabulary				0.031
				[0.007]**
Constant	0.888	0.91	0.478	0.556
	[0.035]**	[0.117]**	[0.185]**	[0.188]**
Wave dummies	YES	YES	YES	YES
Observations	31739	31739	22897	21984
R-squared	0.05	0.05	0.06	0.06

Robust standard errors in brackets

*signicant at 5%; ** signicant at 1%

Table 4: OLS regressions of measurement error on the log of annual earnings, HRS 1992-2004 reports compared to the Box 1 of the W2 tax form.

	m_{it}		
	Total sample	$Y_{it}^a > \$1,000$	$Y_{it}^a > \$10,000$
y_{it}^a	-0.154	-0.085	-0.014
	0.008***	0.005***	0.006**
Constant	1.623	0.925	0.181
	0.077***	0.054***	0.059***
N	26707	26168	21297
R-squared	0.085	0.026	0

Y_{it}^a is the level and y_{it}^a is the log of the Box1 W2 earnings, deflated with the CPI to $2000 dollars, m_{it} is measurement error defined as the log difference between the HRS and the W2 annual earnings; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 5: OLS regressions of measurement error on the log of annual earnings, HRS 1992-2004 reports compared to the Box 1 of the W2 tax form.

	m_{it}					
	Total sample		$Y_{it}^a > \$1,000$		$Y_{it}^a > \$10,000$	
y_{it}^a	-1.262	-2.562	-0.988	-6.137	0.314	-5.62
	0.088***	0.495***	0.084***	0.704***	0.159**	2.244**
$(y_{it}^a)^2$	0.603	2.131	0.465	5.815	-0.157	5.381
	0.045***	0.538***	0.042***	0.712***	0.076**	2.111**
$(y_{it}^a)^3$		-0.581		-1.835		-1.715
		0.192***		0.239***		0.661***
Constant	6.613	10.167	5.258	21.594	-1.526	19.586
	0.427***	1.500***	0.419***	2.310***	0.826*	7.932**
N	26707	26707	26168	26168	21297	21297
R-squared	0.14	0.143	0.04	0.047	0.001	0.002

Y_{it}^a is the level and y_{it}^a is the log of the Box1 W2 earnings, deflated with the CPI to $2000 dollars, m_{it} is measurement error defined as the log difference between the HRS and the W2 annual earnings; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 6: Missing values and zeros in the DER records and the HRS reports, 1992-2004

	Record non-zero	Record zero	Record missing	Total
Report non-zero	26,360	1,790	13,485	41,635
Report zero	4,214	25,691	28,713	58,618
Report missing	1,054	331	2,188	3,573
Total	31,628	27,812	44,386	103,826

Table 7: Properties of measurement error in different subgroups, HRS 1992-2004 compared to the Box 1 of the W2 tax form

	Subgroup	N	σ_m	γ_{m,y^a}
1	Total 1992-2004	26707	0.57	-0.15
2	(1) + 50-65 yrs old, no self employment income	20144	0.49	-0.11
3	(1) + 50-65 yrs old	21871	0.56	-0.15
4	(1) + no self employment income	24632	0.51	-0.12
5	(2) + tenure≥2yrs, full time, salaried	5624	0.31	-0.09
6	(2) + tenure≥2yrs	14520	0.37	-0.07
7	(2) + tenure<2yrs	3125	0.68	-0.20
8	(2) + full time	14277	0.39	-0.12
9	(2) + not full time	5867	0.67	-0.16
10	(2) + salaried	6950	0.37	-0.11
11	(2) + not salaried	10759	0.48	-0.15
12	(2) + financial respondent	13517	0.50	-0.11
13	(2) + not financial respondent	6627	0.46	-0.12
14	(2) + only 1992	4855	0.47	-0.15
15	(2) + only 1994-2004	15289	0.49	-0.10
16	(2) + continuous report	18620	0.49	-0.11
17	(2) + bracketed report	1524	0.45	-0.13
18	(2) + $Y_{it}^a > \$10,000$	16904	0.38	-0.01
19	(2) + outliers dropped	20108	0.49	-0.11
20	(2) + only one W2 record in the year	6262	0.45	-0.09
21	(2) + at least two W2 records in the year	1720	0.56	-0.12
22	(2) + $\sigma_{y_i^a} \leq 0.25$	6430	0.33	-0.02
23	(2) + $0.25 < \sigma_{y_i^a} < 1$	9721	0.47	-0.07
24	(2) + $\sigma_{y^a} \geq 1$	3993	0.69	-0.19

σ_m is the standard deviation of measurement error in a given sub-group; and γ_{m,y^a} is the regression coefficient from an OLS regression of the error on log W2 earnings. Rows 20 and 21 split up the sample based on the number of W2 forms submitted for the same individual in a given year. This variable is only available for a small subset of the sample as a more restrictive consent was needed. Rows 22-25 split up the sample based on the within-individual standard deviation of log administrative earnings in all available years (including years when the biannual HRS was not administered) between 1991-2003.

Table 8: Properties of measurement error in different subgroups by cognition, HRS 1992-2004 compared to the Box 1 of the W2 tax form

		high cognition		low cognition	
	Subgroup	σ_m	γ_{m,y^a}	σ_m	γ_{m,y^a}
1	Total 1992-2004	0.57	-0.16	0.59	-0.18
2	(1) + 50-65 yrs old, no self employment income	0.44	-0.09	0.53	-0.17
3	(1) + 50-65 yrs old	0.56	-0.15	0.57	-0.19
4	(1) + no self employment income	0.46	-0.11	0.55	-0.17
5	(2) + tenure≥2yrs, full time, salaried	0.27	-0.09	0.35	-0.20
6	(2) + tenure≥2yrs	0.34	-0.07	0.40	-0.11
7	(2) + tenure<2yrs	0.64	-0.17	0.70	-0.24
8	(2) + full time	0.36	-0.13	0.42	-0.15
9	(2) + not full time	0.60	-0.11	0.73	-0.24
10	(2) + salaried	0.34	-0.10	0.39	-0.16
11	(2) + not salaried	0.48	-0.12	0.49	-0.17
12	(2) + financial respondent	0.45	-0.09	0.55	-0.17
13	(2) + not financial respondent	0.42	-0.10	0.50	-0.17
14	(2) + only 1992	0.42	-0.14	0.50	-0.18
15	(2) + only 1994-2004	0.44	-0.08	0.55	-0.17
16	(2) + continuous report	0.44	-0.10	0.54	-0.17
17	(2) + bracketed report	0.37	-0.07	0.51	-0.17
18	(2) + $Y^a_{it} > \$10,000$	0.34	-0.02	0.39	-0.03
19	(2) + outliers dropped	0.43	-0.09	0.53	-0.17
20	(2) + only one W2 record in the year	0.38	-0.07	0.50	-0.16
21	(2) + at least two W2 records in the year	0.49	-0.08	0.60	-0.11
22	(2) + $\sigma_{y^a_i} \leq 0.25$	0.31	-0.05	0.31	-0.01
23	(2) + $0.25 < \sigma_{y^a_i} < 1$	0.44	-0.08	0.52	-0.11
24	(2) + $\sigma_{y^a} \geq 1$	0.59	-0.12	0.81	-0.30

σ_m is the standard deviation of measurement error in a given sub-group; and γ_{m,y^a} is the regression coefficient from an OLS regression of the error on log W2 earnings. Rows 20 and 21 split up the sample based on the number of W2 forms submitted for the same individual in a given year. This variable is only available for a small subset of the sample as a more restrictive consent was needed. Rows 22-25 split up the sample based on the within-individual standard deviation of log administrative earnings in all available years (including years when the biannual HRS was not administered) between 1991-2003.

Table 9: Properties of measurement error in different subgroups, $Y_{it}^a > \$1,000$, HRS 1992-2004 compared to the Box 1 of the W2 tax form

	Subgroup	N	σ_m	γ_{m,y^a}
1	Total 1992-2004	26168	0.49	-0.09
2	(1) + 50-65 yrs old, no self employment income	19867	0.44	-0.07
3	(1) + 50-65 yrs old	21507	0.49	-0.09
4	(1) + no self employment income	24207	0.45	-0.07
5	(2) + tenure≥2yrs, full time, salaried	5623	0.30	-0.08
6	(2) + tenure≥2yrs	14479	0.35	-0.04
7	(2) + tenure<2yrs	3046	0.60	-0.14
8	(2) + full time	14248	0.37	-0.08
9	(2) + not full time	5619	0.59	-0.10
10	(2) + salaried	6939	0.36	-0.09
11	(2) + not salaried	10645	0.44	-0.09
12	(2) + financial respondent	13334	0.45	-0.06
13	(2) + not financial respondent	6533	0.42	-0.09
14	(2) + only 1992	4801	0.42	-0.10
15	(2) + only 1994-2004	15066	0.45	-0.06
16	(2) + continuous report	18378	0.44	-0.06
17	(2) + bracketed report	1489	0.41	-0.12
18	(2) + record ≥$10,000 in 2000 dollars	16904	0.38	-0.01
19	(2) + outliers dropped	19833	0.44	-0.07
20	(2) + $Y_{it}^a > \$10,000$	6172	0.40	-0.05
21	(2) + at least two W2 records in the year	1707	0.50	-0.07
22	(2) + $\sigma_{y_i^a} \leq 0.25$	6424	0.32	0.00
23	(2) + $0.25 < \sigma_{y_i^a} < 1$	9651	0.47	-0.07
24	(2) + $\sigma_{y^a} \geq 1$	3792	0.53	-0.11

σ_m is the standard deviation of measurement error in a given sub-group; and γ_{m,y^a} is the regression coefficient from an OLS regression of the error on log W2 earnings. Rows 20 and 21 split up the sample based on the number of W2 forms submitted for the same individual in a given year. This variable is only available for a small subset of the sample as a more restrictive consent was needed. Rows 22-25 split up the sample based on the within-individual standard deviation of log administrative earnings in all available years (including years when the biannual HRS was not administered) between 1991-2003.

Table 10: Properties of measurement error in different subgroups, $Y^a_{it} > \$10,000$, HRS 1992-2004 compared to the Box 1 of the W2 tax form

	Subgroup	N	σ_m	γ_{m,y^a}
1	Total 1992-2004	21297	0.41	-0.01
2	(1) + 50-65 yrs old, no self employment income	16907	0.38	-0.01
3	(1) + 50-65 yrs old	18142	0.41	-0.02
4	(1) + no self employment income	19870	0.38	-0.01
5	(2) + tenure$\geq$2yrs, full time, salaried	5548	0.28	-0.03
6	(2) + tenure$\geq$2yrs	13250	0.32	0.00
7	(2) + tenure$<$2yrs	2070	0.48	-0.08
8	(2) + full time	13584	0.32	-0.01
9	(2) + not full time	3323	0.54	-0.03
10	(2) + salaried	6638	0.32	-0.05
11	(2) + not salaried	8722	0.37	-0.01
12	(2) + financial respondent	11510	0.38	0.00
13	(2) + not financial respondent	5397	0.37	-0.03
14	(2) + only 1992	4129	0.32	-0.02
15	(2) + only 1994-2004	12778	0.39	0.00
16	(2) + continuous report	15812	0.38	-0.01
17	(2) + bracketed report	1095	0.35	-0.05
18	(2) + $Y^a_{it} > \$10,000$	16897	0.38	-0.01
19	(2) + outliers dropped	16882	0.38	-0.01
20	(2) + only one W2 record in the year	5384	0.37	0.01
21	(2) + at least two W2 records in the year	1381	0.40	-0.05
22	(2) + $\sigma_{y^a_i} \leq 0.25$	6102	0.32	0.00
23	(2) + $0.25 < \sigma_{y^a_i} < 1$	7899	0.41	-0.01
24	(2) + $\sigma_{y^a} \geq 1$	2906	0.39	-0.01

σ_m is the standard deviation of measurement error in a given sub-group; and γ_{m,y^a} is the regression coefficient from an OLS regression of the error on log W2 earnings. Rows 20 and 21 split up the sample based on the number of W2 forms submitted for the same individual in a given year. This variable is only available for a small subset of the sample as a more restrictive consent was needed. Rows 22-25 split up the sample based on the within-individual standard deviation of log administrative earnings in all available years (including years when the biannual HRS was not administered) between 1991-2003.

Table 11: Properties of measurement error in different subgroups by cognition, $Y_{it}^a > \$1,000$, HRS 1992-2004 compared to the Box 1 of the W2 tax form

		high cognition		low cognition	
	Subgroup	σ_m	γ_{m,y^a}	σ_m	γ_{m,y^a}
1	Total 1992-2004	0.48	-0.09	0.51	-0.11
2	(1) + 50-65 yrs old, no self employment income	0.40	-0.07	0.46	-0.09
3	(1) + 50-65 yrs old	0.48	-0.09	0.49	-0.10
4	(1) + no self employment income	0.41	-0.07	0.47	-0.09
5	(2) + tenure≥2yrs, full time, salaried	0.27	-0.09	0.30	-0.11
6	(2) + tenure≥2yrs	0.32	-0.06	0.37	-0.06
7	(2) + tenure<2yrs	0.58	-0.13	0.59	-0.14
8	(2) + full time	0.33	-0.09	0.39	-0.09
9	(2) + not full time	0.55	-0.10	0.60	-0.13
10	(2) + salaried	0.33	-0.09	0.35	-0.10
11	(2) + not salaried	0.43	-0.09	0.44	-0.10
12	(2) + financial respondent	0.41	-0.07	0.46	-0.07
13	(2) + not financial respondent	0.39	-0.07	0.45	-0.12
14	(2) + only 1992	0.36	-0.10	0.43	-0.10
15	(2) + only 1994-2004	0.41	-0.06	0.47	-0.08
16	(2) + continuous report	0.40	-0.07	0.46	-0.08
17	(2) + bracketed report	0.36	-0.07	0.45	-0.15
18	(2) + $Y_{it}^a > \$10,000$	0.34	-0.02	0.39	-0.03
19	(2) + outliers dropped	0.40	-0.07	0.46	-0.08
20	(2) + only one W2 record in the year	0.36	-0.06	0.43	-0.07
21	(2) + at least two W2 records in the year	0.47	-0.10	0.52	-0.02
22	(2) + $\sigma_{y_i^a} \leq 0.25$	0.30	-0.03	0.31	0.01
23	(2) + $0.25 < \sigma_{y_i^a} < 1$	0.43	-0.07	0.51	-0.10
24	(2) + $\sigma_{y^a} \geq 1$	0.47	-0.09	0.55	-0.11

σ_m is the standard deviation of measurement error in a given sub-group; and γ_{m,y^a} is the regression coefficient from an OLS regression of the error on log W2 earnings. Rows 20 and 21 split up the sample based on the number of W2 forms submitted for the same individual in a given year. This variable is only available for a small subset of the sample as a more restrictive consent was needed. Rows 22-25 split up the sample based on the within-individual standard deviation of log administrative earnings in all available years (including years when the biannual HRS was not administered) between 1991-2003.

Table 12: Properties of measurement error in different subgroups by cognition, $Y_{it}^{a} > \$10,000$, HRS 1992-2004 compared to the Box 1 of the W2 tax form

		high cognition		low cognition	
	Subgroup	σ_m	γ_{m,y^a}	σ_m	γ_{m,y^a}
1	Total 1992-2004	0.40	-0.03	0.41	-0.03
2	(1) + 50-65 yrs old, no self employment income	0.35	-0.03	0.38	-0.03
3	(1) + 50-65 yrs old	0.41	-0.04	0.41	-0.03
4	(1) + no self employment income	0.35	-0.02	0.39	-0.03
5	(2) + tenure≥2yrs, full time, salaried	0.25	-0.06	0.27	-0.04
6	(2) + tenure≥2yrs	0.29	-0.02	0.34	-0.02
7	(2) + tenure<2yrs	0.49	-0.08	0.47	-0.12
8	(2) + full time	0.31	-0.04	0.34	-0.03
9	(2) + not full time	0.50	-0.03	0.53	-0.09
10	(2) + salaried	0.30	-0.06	0.27	-0.07
11	(2) + not salaried	0.36	-0.01	0.38	-0.04
12	(2) + financial respondent	0.35	-0.03	0.38	-0.02
13	(2) + not financial respondent	0.34	-0.03	0.39	-0.07
14	(2) + only 1992	0.29	-0.06	0.34	-0.03
15	(2) + only 1994-2004	0.36	-0.02	0.40	-0.03
16	(2) + continuous report	0.35	-0.02	0.38	-0.03
17	(2) + bracketed report	0.34	-0.11	0.39	-0.10
18	(2) + $Y_{it}^{a} > \$10,000$	0.35	-0.03	0.38	-0.03
19	(2) + outliers dropped	0.35	-0.03	0.38	-0.03
20	(2) + only one W2 record in the year	0.31	-0.01	0.38	0.01
21	(2) + at least two W2 records in the year	0.39	-0.06	0.45	-0.09
22	(2) + $\sigma_{y_i^a} \leq 0.25$	0.30	-0.03	0.29	0.00
23	(2) + $0.25 < \sigma_{y_i^a} < 1$	0.39	-0.03	0.46	-0.05
24	(2) + $\sigma_{y^a} \geq 1$	0.33	-0.02	0.35	-0.04

σ_m is the standard deviation of measurement error in a given sub-group; and γ_{m,y^a} is the regression coefficient from an OLS regression of the error on log W2 earnings. Rows 20 and 21 split up the sample based on the number of W2 forms submitted for the same individual in a given year. This variable is only available for a small subset of the sample as a more restrictive consent was needed. Rows 22-25 split up the sample based on the within-individual standard deviation of log administrative earnings in all available years (including years when the biannual HRS was not administered) between 1991-2003.

Table 13: Linear regressions of log DER earnings and log error in sample 3

	log DER earnings	log error	log error
log DER earnings			-0.212
			[0.013]**
Education in yrs	0.07	0.008	0.023
	[0.004]**	[0.002]**	[0.002]**
Age in yrs	0.348	-0.064	0.01
	[0.042]**	[0.026]*	[0.024]
Age squared	-0.003	0.001	0
	[0.000]**	[0.000]*	[0.000]
Female	-0.516	-0.042	-0.151
	[0.019]**	[0.009]**	[0.011]**
Hispanic	0.002	-0.046	-0.045
	[0.034]	[0.016]**	[0.016]**
Black	0.025	-0.012	-0.007
	[0.027]	[0.015]	[0.014]
Tenure in yrs	0.026	-0.003	0.003
	[0.001]**	[0.000]**	[0.000]**
Financial respondent	0.083	0.007	0.025
	[0.019]**	[0.008]	[0.008]**
Totall recall	0.069	0.005	0.019
	[0.014]**	[0.007]	[0.007]**
Mental status	0.042	0.014	0.023
	[0.013]**	[0.006]*	[0.006]**
Vocabulary	0.057	0.003	0.015
	[0.011]**	[0.005]	[0.005]**
Constant	-0.383	1.833	1.752
	[1.197]	[0.733]*	[0.680]*
Wave dummies	YES	YES	YES
Observations	17426	17426	17426
R-squared	0.33	0.01	0.13

Standard errors in brackets
*signicant at 5%; ** signicant at 1%

Table 14: Mean reversion in measurement error by six quantiles of within-individual standard deviation of W2 earnings, HRS 1992-2004 compared to the Box 1 of the W2 tax form

	Q1	Q2	Q3	Q4	Q5	Q6
Total sample	-0.029*	-0.056***	-0.058***	-0.081***	-0.154***	-0.25***
$Y^a_{it} > \$1,000$	-0.002	-0.044***	-0.036***	-0.073***	-0.128***	-0.116***
$Y^a_{it} > \$10,000$	0.018	0.005	-0.003	-0.025*	-0.038***	-0.027*

The table contains values of γ_{m,y^a} which is the regression coefficient from an OLS regression of the error on log W2 earnings; The within-individual standard deviation of log W2 earnings use all available years (including years when the biannual HRS was not administered) between 1991-2003; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 15: Mean reversion in measurement error in permanent and transitory earnings by the number of available valid values per person, HRS 1992-2004 compared to the Box 1 of the W2 tax form

	>=2	>=3	>=4	>=5	>=6	>=7
Everybody, N	21827	20043	17370	12506	9316	5656
Permanent earnings	-0.118***	-0.089***	-0.078***	-0.067***	-0.029***	-0.008
Transitory earnings	-0.242***	-0.233***	-0.222***	-0.208***	-0.204***	-0.199***
$Y_{it}^a > \$1,000$, N	21374	19604	16913	12137	9057	5397
Permanent earnings	-0.057***	-0.042***	-0.034***	-0.03***	-0.01	0.013
Transitory earnings	-0.172***	-0.167***	-0.162***	-0.157***	-0.171***	-0.17***
$Y_{it}^a > \$10,000$, N	17171	15511	12952	8852	6277	3535
Permanent earnings	0.001	0.002	0.004	0.019**	0.017*	0.041***
Transitory earnings	-0.155***	-0.155***	-0.168***	-0.159***	-0.188***	-0.196***

The table contains values of γ_{m,y^a} which is the regression coefficient from an OLS regression of the error on log W2 earnings; See the text for the definition of permanent and transitory earnings and the definition of the number of available years; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 16: OLS regressions of log food consumption on permanent (P) and transitory (T) earnings and measurement error in them, HRS 1992-1994 and 2000-2004 reports compared to the Box 1 of the W2 tax form, Restricted to observation with at least 3 available earnings values between 1992-2004

	[1]	[2]	[3]	[4]	[5]	[6]
Error in P	0.235	0.162	0.147	0.191	0.13	0.124
	0.026***	0.023***	0.023***	0.027***	0.023***	0.023***
Error in T	0.028	0.028	0.027	0.026	0.025	0.024
	0.010***	0.010***	0.009***	0.010***	0.010***	0.009**
P W2	0.171	0.167	0.142	-0.75	-0.497	-0.472
	0.009***	0.008***	0.009***	0.122***	0.103***	0.102***
P W2 squared				0.047	0.034	0.032
				0.006***	0.005***	0.005***
T W2	0.024	0.036	0.034	0.036	0.044	0.04
	0.008***	0.008***	0.008***	0.010***	0.010***	0.009***
T W2 squared				0.014	0.011	0.009
				0.005***	0.005**	0.004**
P other income		0.098	0.053		-0.033	-0.061
		0.005***	0.005***		0.016**	0.015***
P other income squared					0.008	0.007
					0.001***	0.001***
T other income		0.063	0.04		0.047	0.026
		0.005***	0.004***		0.005***	0.004***
T other income squared					0.003	0.002
					0.001***	0.001***
No other income		0.512	0.358		0.278	0.159
		0.044***	0.041***		0.044***	0.042***
Wealth controls		Y	Y		Y	Y
Demographic controls			Y			Y
Year dummies	Y	Y	Y	Y	Y	Y
N	13684	13684	13684	13684	13684	13684
R-squared	0.079	0.18	0.231	0.09	0.193	0.241

For people with no-other household income, 0 is imputed and a separate dummy “No other income” indicates these cases; Wealth controls include log household financial wealth, log household non-financial wealth and dummies indicating the lack of such wealth items; Demographics controls include singleness, years of education, gender, Hispanic and black race; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 17: OLS regressions of log food consumption on permanent (P) and transitory (T) earnings and measurement error in them, HRS 1992-1994 and 2000-2004 reports compared to the Box 1 of the W2 tax form, Restricted to observation with at least 6 available earnings values between 1992-2004

	[1]	[2]	[3]	[4]	[5]	[6]
Error in P	0.249	0.159	0.149	0.227	0.15	0.145
	0.052***	0.049***	0.048***	0.053***	0.048***	0.047***
Error in T	0.026	0.027	0.028	0.023	0.023	0.025
	0.016	0.016*	0.015*	0.017	0.016	0.015
P W2	0.198	0.199	0.174	-0.95	-0.507	-0.403
	0.015***	0.015***	0.016***	0.266***	0.254**	0.239*
P W2 squared				0.058	0.036	0.029
				0.013***	0.013***	0.012**
T W2	0.02	0.04	0.039	0.032	0.047	0.047
	0.011*	0.012***	0.011***	0.013**	0.013***	0.013***
T W2 squared				0.011	0.008	0.008
				0.007	0.007	0.006
P other income		0.102	0.057		-0.026	-0.063
		0.007***	0.008***		0.029	0.028**
P other income squared					0.008	0.008
					0.002***	0.002***
T other income		0.068	0.047		0.055	0.036
		0.007***	0.006***		0.006***	0.006***
T other income squared					0.002	0.001
					0.001*	0.001
No other income		0.523	0.395		0.341	0.234
		0.065***	0.061***		0.061***	0.059***
Wealth controls		Y	Y		Y	Y
Demographic controls			Y			Y
Year dummies	Y	Y	Y	Y	Y	Y
N	13684	13684	13684	13684	13684	13684
R-squared	0.079	0.18	0.231	0.09	0.193	0.241

For people with no-other household income, 0 is imputed and a separate dummy “No other income” indicates these cases; Wealth controls include log household financial wealth, log household non-financial wealth and dummies indicating the lack of such wealth items; Demographics controls include singleness, years of education, gender, Hispanic and black race; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 18: OLS regressions of log food consumption on permanent (P) and transitory (T) earnings and measurement error in them, HRS 1992-1994 and 2000-2004 reports compared to the Box 1 of the W2 tax form, Restricted to $Y_{it}^{a} > \$1,000$ and observation with at least 3 available earnings values between 1992-2004

	[1]	[2]	[3]	[4]	[5]	[6]
Error in P	0.232	0.146	0.134	0.208	0.131	0.126
	0.031***	0.027***	0.027***	0.031***	0.027***	0.026***
Error in T	0.024	0.025	0.024	0.022	0.022	0.022
	0.011**	0.011**	0.010**	0.011**	0.011**	0.010**
P W2	0.182	0.177	0.151	-1.241	-0.77	-0.772
	0.010***	0.009***	0.010***	0.161***	0.154***	0.144***
P W2 squared				0.072	0.048	0.047
				0.008***	0.008***	0.007***
T W2	0.027	0.044	0.04	0.042	0.051	0.046
	0.010***	0.010***	0.009***	0.011***	0.011***	0.010***
T W2 squared				0.033	0.019	0.017
				0.010***	0.010**	0.009*
P other income		0.098	0.053		-0.029	-0.057
		0.005***	0.005***		0.016*	0.015***
P other income squared					0.008	0.007
					0.001***	0.001***
T other income		0.063	0.04		0.047	0.027
		0.005***	0.005***		0.005***	0.004***
T other income squared					0.003	0.002
					0.001***	0.001***
No other income		0.513	0.36		0.283	0.165
		0.044***	0.042***		0.045***	0.043***
Wealth controls		Y	Y		Y	Y
Demographic controls			Y			Y
Year dummies	Y	Y	Y	Y	Y	Y
N	13368	13368	13368	13368	13368	13368
R-squared	0.079	0.179	0.229	0.092	0.191	0.239

For people with no-other household income, 0 is imputed and a separate dummy "No other income" indicates these cases; Wealth controls include log household financial wealth, log household non-financial wealth and dummies indicating the lack of such wealth items; Demographics controls include singleness, years of education, gender, Hispanic and black race; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 19: OLS regressions of log food consumption on permanent (P) and transitory (T) earnings and measurement error in them, HRS 1992-1994 and 2000-2004 reports compared to the Box 1 of the W2 tax form, Restricted to $Y_{it}^{a} > \$1,000$ and observation with at least 6 available earnings values between 1992-2004

	[1]	[2]	[3]	[4]	[5]	[6]
Error in P	0.233	0.129	0.116	0.219	0.128	0.118
	0.060***	0.055**	0.053**	0.060***	0.054**	0.052**
Error in T	0.027	0.025	0.028	0.024	0.023	0.026
	0.018	0.017	0.017*	0.018	0.017	0.017
P W2	0.21	0.208	0.183	-1.147	-0.661	-0.56
	0.016***	0.016***	0.017***	0.295***	0.276**	0.248**
P W2 squared				0.068	0.043	0.037
				0.015***	0.014***	0.012***
T W2	0.027	0.054	0.054	0.039	0.057	0.056
	0.014**	0.014***	0.013***	0.015**	0.015***	0.014***
T W2 squared				0.024	0.012	0.009
				0.015	0.014	0.013
P other income		0.103	0.058		-0.021	-0.059
		0.007***	0.008***		0.029	0.028**
P other income squared					0.008	0.007
					0.002***	0.002***
T other income		0.069	0.048		0.056	0.036
		0.007***	0.007***		0.006***	0.006***
T other income squared					0.001	0.001
					0.001	0.001
No other income		0.528	0.403		0.358	0.249
		0.065***	0.061***		0.061***	0.059***
Wealth controls		Y	Y		Y	Y
Demographic controls			Y			Y
Year dummies	Y	Y	Y	Y	Y	Y
N	6355	6355	6355	6355	6355	6355
R-squared	0.095	0.197	0.241	0.103	0.206	0.249

For people with no-other household income, 0 is imputed and a separate dummy "No other income" indicates these cases; Wealth controls include log household financial wealth, log household non-financial wealth and dummies indicating the lack of such wealth items; Demographics controls include singleness, years of education, gender, Hispanic and black race; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 20: OLS regressions of log food consumption on permanent (P) and transitory (T) earnings and measurement error in them, HRS 1992-1994 and 2000-2004 reports compared to the Box 1 of the W2 tax form, Restricted to $Y_{it}^{a} > \$10,000$ and observation with at least 3 available earnings values between 1992-2004

	[1]	[2]	[3]	[4]	[5]	[6]
Error in P	0.181	0.102	0.1	0.183	0.101	0.1
	0.041***	0.035***	0.034***	0.041***	0.035***	0.034***
Error in T	0.021	0.019	0.021	0.022	0.018	0.019
	0.014	0.014	0.013	0.014	0.013	0.013
P W2	0.271	0.249	0.232	-0.662	-0.205	-0.302
	0.014***	0.015***	0.016***	0.380*	0.36	0.332
P W2 squared				0.044	0.022	0.025
				0.018**	0.017	0.016
T W2	0.062	0.088	0.077	0.078	0.09	0.081
	0.019***	0.019***	0.018***	0.019***	0.019***	0.019***
T W2 squared				0.106	0.046	0.052
				0.037***	0.036	0.035
P other income		0.093	0.052		-0.01	-0.041
		0.005***	0.005***		0.017	0.016**
P other income squared					0.007	0.006
					0.001***	0.001***
T other income		0.06	0.038		0.045	0.025
		0.005***	0.005***		0.005***	0.005***
T other income squared					0.003	0.002
					0.001***	0.001***
No other income		0.462	0.327		0.256	0.145
		0.046***	0.043***		0.046***	0.044***
Wealth controls		Y	Y		Y	Y
Demographic controls			Y			Y
Year dummies	Y	Y	Y	Y	Y	Y
N	10514	10514	10514	10514	10514	10514
R-squared	0.09	0.182	0.23	0.092	0.188	0.235

For people with no-other household income, 0 is imputed and a separate dummy “No other income” indicates these cases; Wealth controls include log household financial wealth, log household non-financial wealth and dummies indicating the lack of such wealth items; Demographics controls include singleness, years of education, gender, Hispanic and black race; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 21: OLS regressions of log food consumption on permanent (P) and transitory (T) earnings and measurement error in them, HRS 1992-1994 and 2000-2004 reports compared to the Box 1 of the W2 tax form, Restricted to $Y_{it}^{a} > \$10,000$ and observation with at least 6 available earnings values between 1992-2004

	[1]	[2]	[3]	[4]	[5]	[6]
Error in P	0.246	0.132	0.134	0.248	0.134	0.133
	0.091***	0.084	0.083	0.091***	0.083	0.082
Error in T	0.025	0.012	0.016	0.025	0.011	0.016
	0.029	0.028	0.027	0.029	0.028	0.027
P W2	0.284	0.263	0.26	-0.295	-0.086	0.082
	0.024***	0.024***	0.027***	0.654	0.582	0.565
P W2 squared				0.027	0.017	0.008
				0.031	0.028	0.027
T W2	0.029	0.059	0.05	0.037	0.056	0.05
	0.027	0.027**	0.026*	0.028	0.027**	0.027*
T W2 squared				0.045	-0.007	0.006
				0.053	0.051	0.05
P other income		0.095	0.055		0.003	-0.032
		0.008***	0.008***		0.03	0.029
P other income squared					0.006	0.006
					0.002***	0.002***
T other income		0.065	0.044		0.055	0.035
		0.007***	0.007***		0.007***	0.007***
T other income squared					0.002	0.001
					0.001*	0.001
No other income		0.469	0.345		0.314	0.207
		0.069***	0.065***		0.069***	0.067***
Wealth controls		Y	Y		Y	Y
Demographic controls			Y			Y
Year dummies	Y	Y	Y	Y	Y	Y
N	4399	4399	4399	4399	4399	4399
R-squared	0.105	0.202	0.244	0.106	0.206	0.248

For people with no-other household income, 0 is imputed and a separate dummy “No other income” indicates these cases; Wealth controls include log household financial wealth, log household non-financial wealth and dummies indicating the lack of such wealth items; Demographics controls include singleness, years of education, gender, Hispanic and black race; *, ** and *** denote significance at 10, 5 and 1 percent level respectively; robust standard errors are clustered on the household level.

Table 22: Dynamic earnings models, Job-switchers excluded, 5000 burnt and 5000 real draws

	Xtreg	WinBUGS						
	RE	model 0	model 1	model 2	model 3	model 4	model 5	model 6
Constant	10.574	10.636	10.667	10.619	10.631	10.656	10.622	10.59
	[0.018]**	[0.053]**	[0.067]**	[0.038]**	[0.063]**	[0.060]**	[0.032]**	[0.038]**
Age 49-52				reference category				
Age 53-56	0.029	-0.043	-0.079	-0.04	-0.029	-0.105	-0.083	-0.043
	[0.009]**	[0.066]	[0.078]	[0.059]	[0.058]	[0.057]	[0.041]*	[0.059]
Age 57-60	0.02	0.112	-0.008	0.02	-0.01	-0.029	0.041	0.074
	[0.015]	[0.064]	[0.094]	[0.037]	[0.065]	[0.076]	[0.057]	[0.044]
Age 61-64	-0.073	-0.072	-0.116	-0.059	-0.078	-0.105	-0.075	-0.029
	[0.021]**	[0.057]	[0.071]	[0.034]	[0.074]	[0.061]	[0.029]**	[0.039]
Years of education	0.084	0.083	0.081	0.081	0.078	0.078	0.077	0.078
	[0.005]**	[0.005]**	[0.004]**	[0.003]**	[0.004]**	[0.004]**	[0.003]**	[0.005]**
Black	-0.133	-0.137	-0.161	-0.18	-0.164	-0.18	-0.107	-0.137
	[0.048]**	[0.049]**	[0.052]**	[0.033]**	[0.044]**	[0.043]**	[0.032]**	[0.037]**
Hispanic	-0.187	-0.176	-0.191	-0.185	-0.222	-0.225	-0.238	-0.219
	[0.059]**	[0.052]**	[0.055]**	[0.027]**	[0.047]**	[0.059]**	[0.027]**	[0.066]**
1992	-0.017	-0.021	-0.009	-0.009	-0.009	-0.009	-0.01	-0.009
	[0.012]	[0.011]	[0.008]	[0.007]	[0.008]	[0.008]	[0.008]	[0.008]
1993	-0.007	-0.015	-0.007	-0.006	-0.006	-0.007	-0.007	-0.006
	[0.012]	[0.011]	[0.009]	[0.009]	[0.009]	[0.010]	[0.010]	[0.009]
1994	-0.014	-0.024	-0.01	-0.007	-0.009	-0.009	-0.009	-0.008
	[0.013]	[0.012]*	[0.012]	[0.011]	[0.011]	[0.012]	[0.012]	[0.012]
1995	-0.011	-0.028	-0.015	-0.012	-0.014	-0.015	-0.014	-0.012
	[0.013]	[0.012]*	[0.013]	[0.013]	[0.011]	[0.013]	[0.014]	[0.013]
1996	-0.008	-0.026	-0.021	-0.017	-0.021	-0.021	-0.019	-0.017
	[0.015]	[0.013]*	[0.014]	[0.015]	[0.013]	[0.015]	[0.015]	[0.015]
1997	0.013	-0.012	-0.007	-0.001	-0.006	-0.008	-0.004	-0.002
	[0.016]	[0.012]	[0.015]	[0.015]	[0.014]	[0.016]	[0.016]	[0.016]
1998	0.028	-0.001	-0.009	-0.002	-0.005	-0.009	-0.004	-0.002
	[0.017]	[0.013]	[0.016]	[0.017]	[0.014]	[0.017]	[0.016]	[0.017]
1999	0.019	-0.016	-0.02	-0.012	-0.015	-0.02	-0.014	-0.013
	[0.018]	[0.013]	[0.018]	[0.017]	[0.015]	[0.018]	[0.017]	[0.017]
2000	0.031	-0.005	-0.026	-0.017	-0.021	-0.026	-0.018	-0.017
	[0.020]	[0.014]	[0.019]	[0.020]	[0.017]	[0.020]	[0.018]	[0.018]
2001	0.022	-0.018	-0.036	-0.026	-0.03	-0.036	-0.028	-0.026
	[0.020]	[0.014]	[0.019]	[0.021]	[0.018]	[0.021]	[0.019]	[0.019]
2002	0.025	-0.016	-0.039	-0.031	-0.033	-0.04	-0.031	-0.03
	[0.022]	[0.015]	[0.020]	[0.023]	[0.020]	[0.021]	[0.021]	[0.021]
2003	0.018	-0.025	-0.044	-0.036	-0.039	-0.045	-0.036	-0.034
	[0.023]	[0.015]	[0.021]*	[0.024]	[0.021]	[0.022]*	[0.022]	[0.023]
V(persistent, 1991)	0.57	0.572	0.51	0.513	0.511	0.51	0.512	0.513
	[0.011]**	[0.011]**	[0.011]**	[0.011]**	[0.010]**	[0.011]**	[0.012]**	[0.011]**
V(transitory)	0.228	0.23	0.096	0.072	0.079	0.098	0.069	0.085
	[0.002]**	[0.002]**	[0.003]**	[0.005]**	[0.006]**	[0.003]**	[0.007]**	[0.006]**
V(persistent)			0.169	0.175	0.175	0.167	0.176	0.174
			[0.003]**	[0.003]**	[0.003]**	[0.004]**	[0.003]**	[0.004]**
MA(1) term				-0.419	-0.241		-0.496	-0.127
				[0.098]**	[0.115]*		[0.157]**	[0.071]
MA(2) term					0.118			0.153
					[0.053]*			[0.047]**
AR(1) term						1.006	0.998	0.998
						[0.007]**	[0.007]**	[0.007]**

Table 23: Dynamic earnings models, Job-switchers included, 3000 burnt and 3000 real draws

	Xtreg	WinBUGS						
	RE	model 0	model 1	model 2	model 3	model 4	model 5	model 6
Constant	10.54	10.979	11.071	11.127				
	[0.027]**	[0.068]**	[0.153]**	[0.084]**				
Age 49-52				reference category				
Age 53-56	0.05	-0.078	-0.27	-0.347				
	[0.018]**	[0.079]	[0.148]	[0.081]**				
Age 57-60	0.01	-0.119	-0.335	-0.431				
	[0.025]	[0.080]	[0.178]	[0.105]**				
Age 61-64	-0.347	-0.505	-0.602	-0.658				
	[0.033]**	[0.069]**	[0.149]**	[0.085]**				
Years of education	0.092	0.089	0.082	0.084				
	[0.006]**	[0.006]**	[0.006]**	[0.007]**				
Black	-0.131	-0.137	-0.2	-0.142				
	[0.058]*	[0.059]*	[0.062]**	[0.052]**				
Hispanic	-0.149	-0.162	-0.25	-0.231				
	[0.067]*	[0.064]*	[0.089]**	[0.072]**				
1992	-0.022	-0.048	-0.037	-0.037				
	[0.027]	[0.028]	[0.016]*	[0.016]*				
1993	-0.094	-0.146	-0.123	-0.124				
	[0.027]**	[0.028]**	[0.020]**	[0.022]**				
1994	-0.092	-0.163	-0.158	-0.156				
	[0.028]**	[0.028]**	[0.024]**	[0.026]**				
1995	-0.078	-0.176	-0.187	-0.181				
	[0.029]**	[0.029]**	[0.027]**	[0.029]**				
1996	-0.098	-0.226	-0.25	-0.24				
	[0.030]**	[0.029]**	[0.031]**	[0.032]**				
1997	-0.11	-0.268	-0.293	-0.284				
	[0.032]**	[0.030]**	[0.033]**	[0.035]**				
1998	-0.108	-0.297	-0.324	-0.318				
	[0.033]**	[0.029]**	[0.036]**	[0.038]**				
1999	-0.154	-0.368	-0.411	-0.403				
	[0.034]**	[0.030]**	[0.040]**	[0.040]**				
2000	-0.175	-0.418	-0.48	-0.467				
	[0.036]**	[0.030]**	[0.042]**	[0.042]**				
2001	-0.206	-0.468	-0.551	-0.536				
	[0.037]**	[0.031]**	[0.045]**	[0.044]**				
2002	-0.219	-0.503	-0.629	-0.609				
	[0.039]**	[0.033]**	[0.048]**	[0.047]**				
2003	-0.274	-0.571	-0.726	-0.7				
	[0.041]**	[0.033]**	[0.051]**	[0.049]**				
V(persistent, 1991)	0.744	0.75	0.623	0.577				
	[0.014]**	[0.014]**	[0.014]**	[0.014]**				
V(transitory)	0.648	0.655	0.214	0.315				
	[0.004]**	[0.004]**	[0.007]**	[0.008]**				
V(persistent)			0.478	0.417				
			[0.006]**	[0.008]**				
MA(1) term				0.405				
				[0.020]**				
MA(2) term								
AR(1) term								

Figure 1: Histogram of measurement error (log survey report minus log DER record) in sample 3

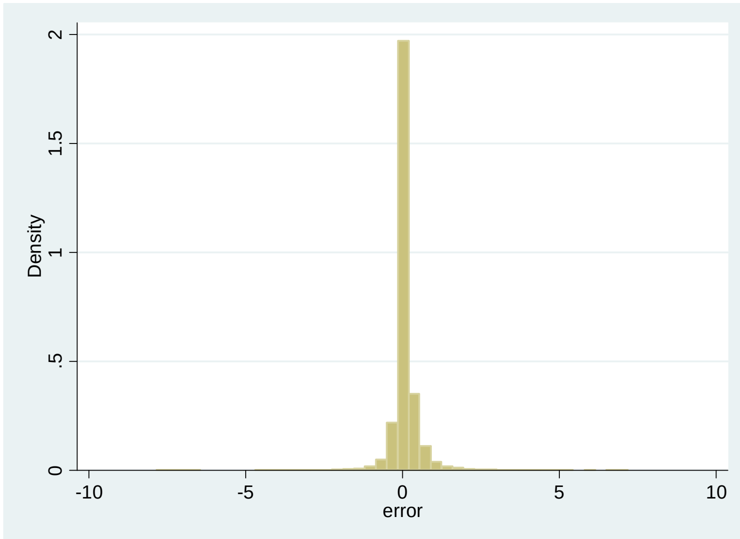


Figure 2: Histogram of measurement error (log survey report minus log DER record) between -2 and 2 in sample 3

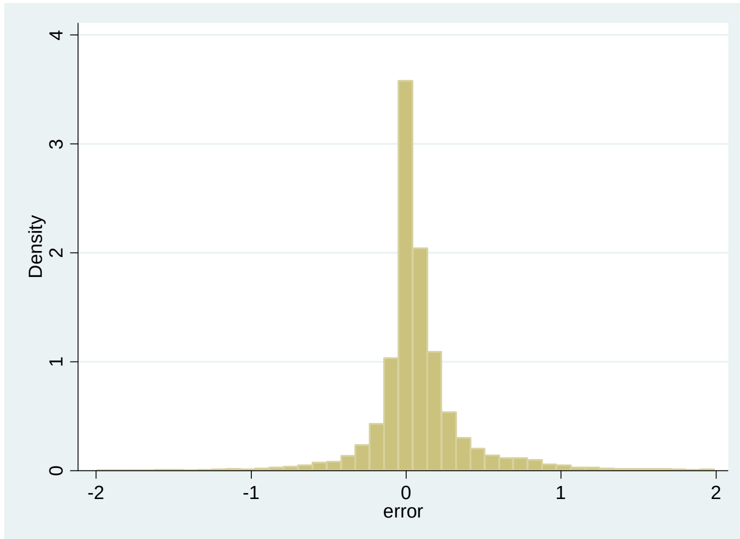


Figure 3: Non-parametric regression of of measurement error on DER earnings in sample 3, with scatter plot, restricted to DER>5, HRS 1992-2004 compared to the Box 1 of the W2 tax form

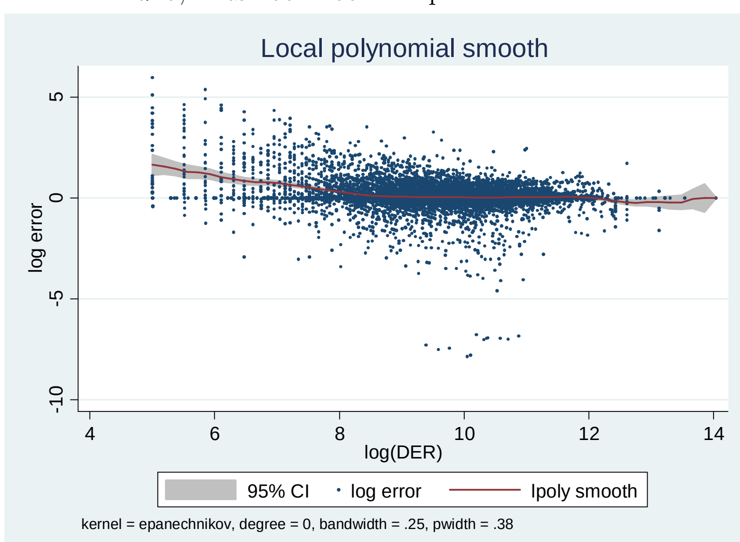


Figure 4: Non-parametric regression of of measurement error on DER earnings in sample 3, without scatter plot, restricted DER>7, HRS 1992-2004 compared to the Box 1 of the W2 tax form

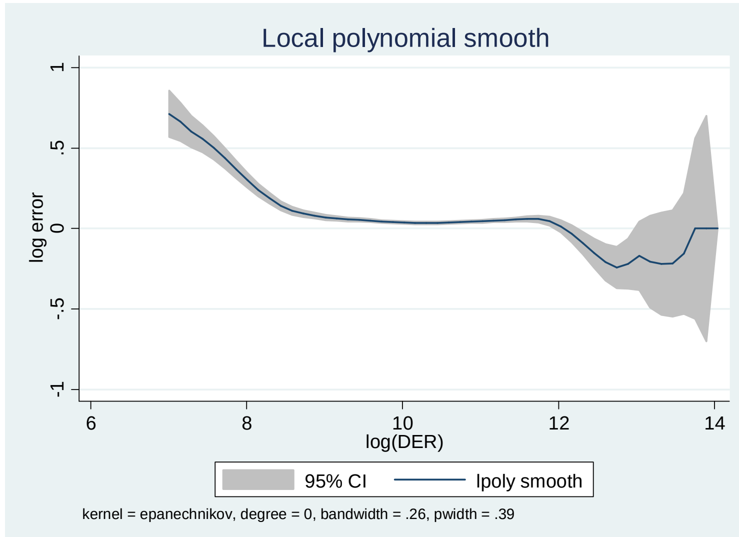
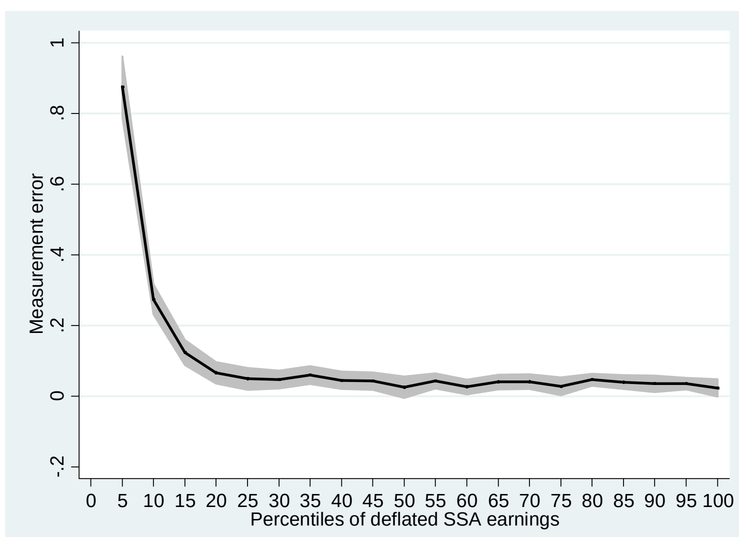


Figure 5: Regression of measurement error on quantiles of log annual earnings in sample 3, HRS 1992-2004 compared to the Box 1 of the W2 tax form



The horizontal axis shows 20 equal sized quantiles of y_{it}^{a} and the vertical axis shows the average of m_{it} within each quantiles; the gray area shows 95 percent confidence intervals using robust standard errors that are clustered on the household level.

Figure 6: Sensitivity analysis for the role of α_c on $\mathbb{E}_{c,z,x|y^a}[m|y^a]$

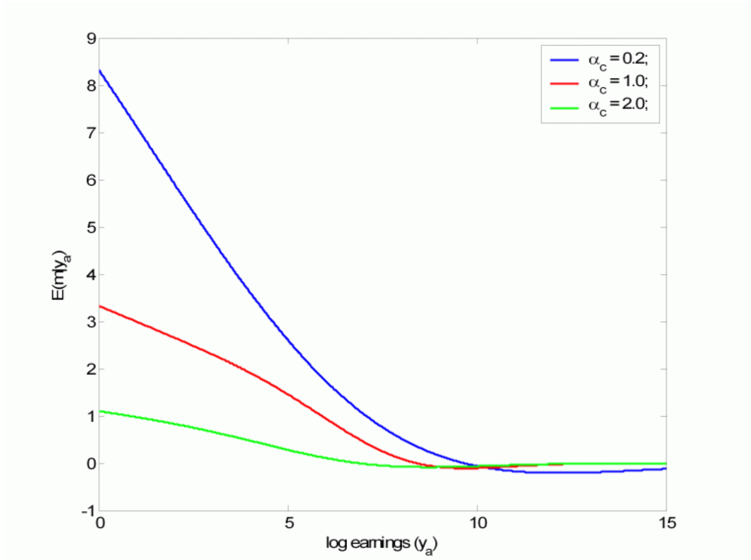


Figure 7: Sensitivity analysis for the role of σ_c^2 on $\mathbb{E}_{c,z,x|y^a}[m|y^a]$

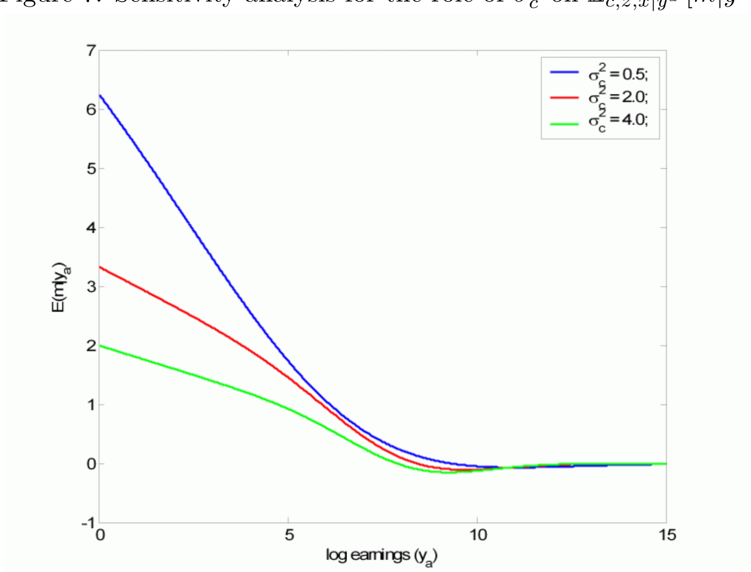


Figure 8: Sensitivity analysis for the role of γ_1 on $\mathbb{E}_{c,z,x|y^a}[m|y^a]$

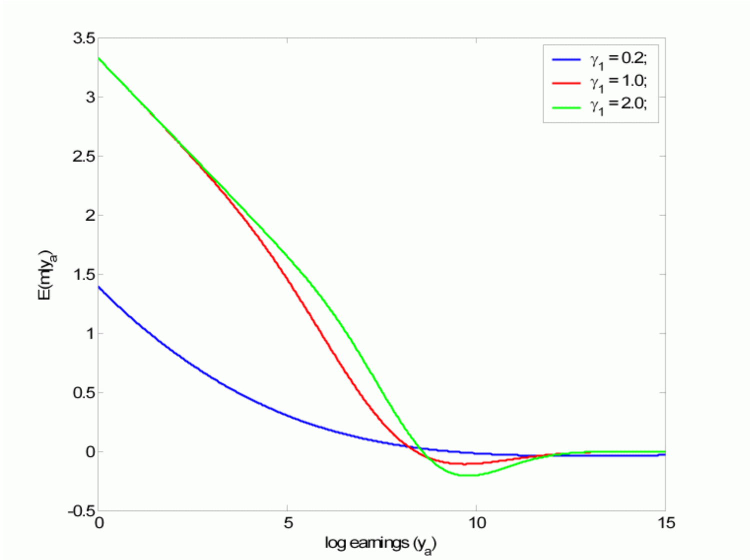


Figure 9: Sensitivity analysis for the role of γ_1 on $T(c)$

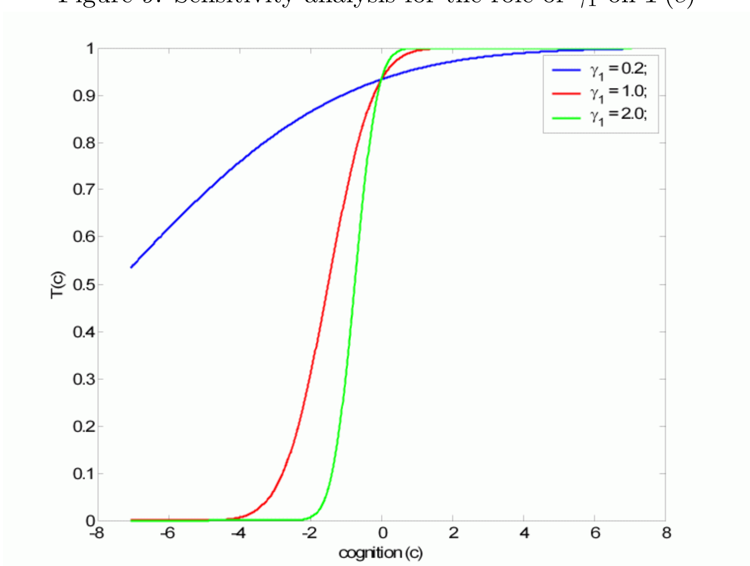


Figure 10: Histogram of measurement error in the simulated data

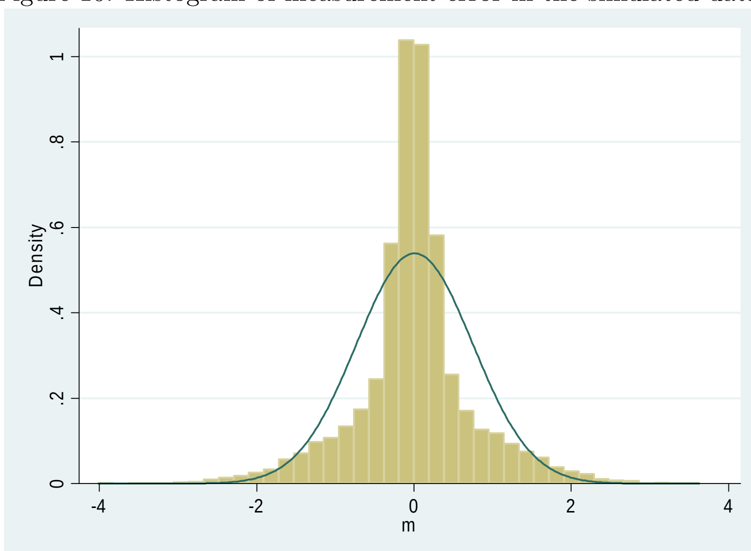
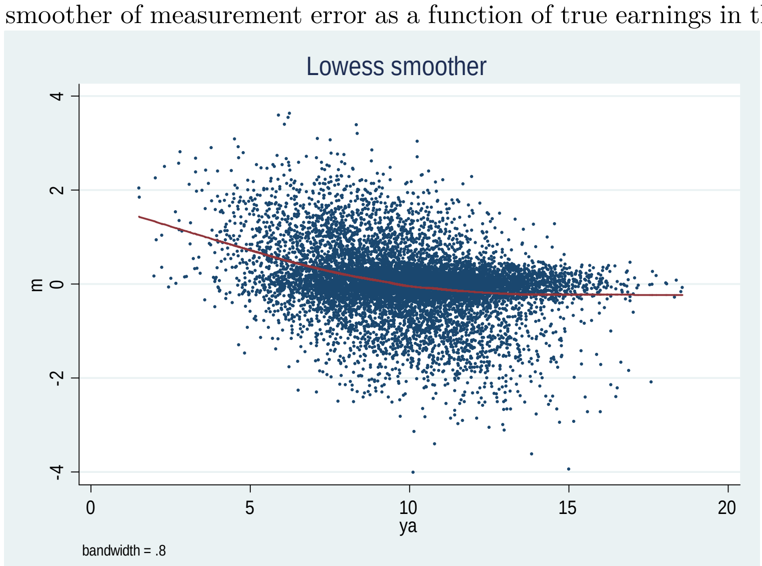


Figure 11: Lowess smoother of measurement error as a function of true earnings in the simulated data



Appendix B: Proofs from Section 4

Proof of theorem 1

$$\begin{aligned}
\Pr\left(g=3|y_{it}^{a}\right) &= \frac{f\left(y_{it}^{a}|g=3\right)\Pr\left(g=3\right)}{f\left(y_{it}^{a}\right)} = \frac{\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}}{\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}+\frac{1}{\sigma}\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)\frac{p_1}{p_1+p_3}} \\
&= \frac{1}{1+\frac{\frac{1}{\sigma}\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)\frac{p_1}{p_1+p_3}}{\frac{1}{\sigma^{3,T}}\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\frac{p_3}{p_1+p_3}}} = \frac{1}{1+f\left(y_{it}^{a}\right)}
\end{aligned}$$

The function $\frac{1}{1+f\left(y_{it}^{a}\right)}$ is decreasing and convex, so in order to prove the theorem I need that $f\left(y_{it}^{a}\right)$ is increasing and convex. In the following derivation I use the well known fact that $\phi'\left(x\right)/\phi\left(x\right)=-x$

$$\begin{aligned}
\frac{\partial\frac{\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)}{\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)}}{\partial y_{it}^{a}} &= \frac{\phi'\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)-\phi'\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)}{\left[\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)\right]^{2}} \\
&= \frac{\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)}{\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)}\left[\frac{\phi'\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)}{\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)}-\frac{\phi'\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)}{\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)}\right] \\
&= \frac{\phi\left(\frac{y_{it}^{a}-\mu}{\sigma}\right)}{\phi\left(\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}\right)}\left[\frac{y_{it}^{a}-\mu^{3,T}}{\sigma^{3,T}}-\frac{y_{it}^{a}-\mu}{\sigma}\right]
\end{aligned}$$

The bracketed value is positive, because $\mu^{3,T}<\mu$ and $\sigma^{3,T}<\sigma$. Thus, $f\left(y_{it}^{a}\right)$ is an increasing function. The second derivative will also be positive, because it is the product of two increasing function. Thus, $\Pr\left(g=3|y_{it}^{a}\right)$ and $E\left(m_{it}|y_{it}^{a}\right)$ is a decreasing and convex function.

Proof of Theorem 2

$$
\begin{aligned}
\mathbb{E}(m) &= \mathbb{E}\left((\tau_p-1)\,y^t+\upsilon|y^t>0\right)\mathbb{P}\left(y^t>0\right)+\mathbb{E}\left((\tau_n-1)\,y^t+\upsilon|y^t\leq 0\right)\mathbb{P}\left(y^t\leq 0\right)\\
&= (\tau_p-1)\,\mathbb{E}\left(y^t|y^t>0\right)\mathbb{P}\left(y^t>0\right)+(\tau_n-1)\,\mathbb{E}\left(y^t|y^t\leq 0\right)\mathbb{P}\left(y^t\leq 0\right)\\
&= \frac{\tau_p-1}{2}\mathbb{E}\left(y^t|y^t>0\right)+\frac{\tau_n-1}{2}\mathbb{E}\left(y^t|y^t\leq 0\right)\\
&= \frac{\tau_p-1}{2}\sigma_t\frac{\phi(0)}{1-\Phi(0)}+\frac{\tau_n-1}{2}\frac{-\phi(0)}{\Phi(0)}\\
&= \frac{\tau_p-\tau_n}{4}\sigma_t\phi(0)\approx 0.1\sigma_t\left(\tau_p-\tau_n\right)
\end{aligned}
$$

Proof of Theorem 3

$$
\begin{aligned}
\mathbb{E}(m|y^a) &= \mathbb{E}\left((\tau_p-1)\,y^t+\upsilon|y^t>0,y^a\right)\mathbb{P}\left(y^t>0|y^a\right)\\
&\quad +\mathbb{E}\left((\tau_n-1)\,y^t+\upsilon|y^t\leq 0,y^a\right)\mathbb{P}\left(y^t\leq 0|y^a\right)\\
&= (\tau_p-1)\,\mathbb{E}\left(y^t|y^t>0,y^a\right)\mathbb{P}\left(y^t>0|y^a\right)\\
&\quad +(\tau_n-1)\,\mathbb{E}\left(y^t|y^t\leq 0,y^a\right)\mathbb{P}\left(y^t\leq 0|y^a\right)\\
&= *
\end{aligned}
$$

The distribution of $y^t|y^a$ is normal with

$$
\begin{aligned}
\mathbb{E}\left(y^t|y^a\right) &= \frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a-\alpha\right)\\
\mathbb{V}\left(y^t|y^a\right) &= \sigma_t^2-\frac{\sigma_t^4}{\sigma_t^2+\sigma_p^2}=\frac{\sigma_t^2\left(\sigma_t^2+\sigma_p^2\right)-\sigma_t^4}{\sigma_t^2+\sigma_p^2}\\
&= \frac{\sigma_t^2\sigma_p^2}{\sigma_t^2+\sigma_p^2}
\end{aligned}
$$

Thus

$$
\begin{aligned}
* \quad = \quad & (\tau_p - 1)\,\mathbb{E}\left(y^t | y^t > 0, y^a\right) \mathbb{P}\left(\frac{y^t - \frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right)}{\frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}}} > - \frac{\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right)}{\frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}}} \middle| y^a \right) \\
& + (\tau_n - 1)\,\mathbb{E}\left(y^t | y^t \leq 0, y^a\right) \mathbb{P}\left(\frac{y^t - \frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right)}{\frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}}} \leq - \frac{\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right)}{\frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}}} \middle| y^a \right) \\
= \quad & (\tau_p - 1)\,\mathbb{E}\left(y^t | y^t > 0, y^a\right) \Phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \right) \\
& + (\tau_n - 1)\,\mathbb{E}\left(y^t | y^t \leq 0, y^a\right) \Phi\left(\frac{-\sigma_t\left(y^a - \alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \right) \\
= \quad & (\tau_p - 1)\left(\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right) + \frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}} \frac{\phi\left(\frac{\sigma_t(y^a-\alpha)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)}{1 - \Phi\left(\frac{-\sigma_t(y^a-\alpha)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)} \right) \Phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \right) \\
& + (\tau_n - 1)\left(\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right) + \frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}} \frac{-\phi\left(\frac{\sigma_t(y^a-\alpha)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)}{\Phi\left(\frac{-\sigma_t(y^a-\alpha)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)} \right) \Phi\left(\frac{-\sigma_t\left(y^a - \alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \right) \\
= \quad & (\tau_p - \tau_n)\left[\frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}}\phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \right) + \frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right)\Phi\left(\frac{\sigma_t\left(y^a - \alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \right) \right] \\
& + (\tau_n - 1)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a - \alpha\right)
\end{aligned}
$$

Proof of Theorem 4

$$
\begin{aligned}
\frac{\partial}{\partial y^a}\mathbb{E}\left(m|y^a\right) &= \left(\tau_n-1\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2} \\
&\quad +\left(\tau_p-\tau_n\right)\frac{\sigma_t\sigma_p}{\sqrt{\sigma_t^2+\sigma_p^2}}\phi'\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\frac{\sigma_t}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}} \\
&\quad +\left(\tau_p-\tau_n\right)\left[\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\lneqq\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\right] \\
&\quad +\left(\tau_p-\tau_n\right)\left[\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a-\alpha\right)\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\frac{\sigma_t}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right] \\
&= \left(\tau_n-1\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2} \\
&\quad +\left(\tau_p-\tau_n\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left[\phi'\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)+\Phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\right] \\
&\quad +\left(\tau_p-\tau_n\right)\left[\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a-\alpha\right)\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\frac{\sigma_t}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right] \\
&= \left(\tau_n-1\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2} \\
&\quad +\left(\tau_p-\tau_n\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left[\frac{\phi'\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)}{\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)}\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)+\Phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\right] \\
&\quad +\left(\tau_p-\tau_n\right)\left[\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a-\alpha\right)\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\frac{\sigma_t}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right] \\
&= \left(\tau_n-1\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2} \\
&\quad +\left(\tau_p-\tau_n\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left[\left(-\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)+\Phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\right] \\
&\quad +\left(\tau_p-\tau_n\right)\left[\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\left(y^a-\alpha\right)\phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)\frac{\sigma_t}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right] \\
&= \left(\tau_n-1\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}+\left(\tau_p-\tau_n\right)\frac{\sigma_t^2}{\sigma_t^2+\sigma_p^2}\Phi\left(\frac{\sigma_t\left(y^a-\alpha\right)}{\sigma_p\sqrt{\sigma_t^2+\sigma_p^2}}\right)
\end{aligned}
$$

Proof of Theorem 6

First I apply the law of iterated expectations:

$$\begin{aligned}
\mathbb{E}_{c,z,x|y^a}\left[m|y^a\right] &= \mathbb{E}_{c|y^a}\left[\mathbb{E}_{z,x|y^a,c}\left[\left[\tau\left(c,z\right)-1\right]y^t|y^a,c\right]|y^a\right] \\
&= \mathbb{E}_{c|y^a}\left[\left(\mathbb{E}_{z|c}\left[\tau\left(c,z\right)|c\right]-1\right)\mathbb{E}_{x|y^a,c}\left[y^t|y^a,c\right]|y^a\right] \\
&= \mathbb{E}_{c|y^a}\left[\left(T\left(c\right)-1\right)\mathbb{E}_{x|y^a,c}\left[y^t|y^a,c\right]|y^a\right] = *
\end{aligned}$$

Note the normality assumption implies that

$$\begin{aligned}
\mathbb{E}_{x|y^a,c}\left[y^t|y^a,c\right] &= \begin{bmatrix} \sigma_t^2 & 0 \end{bmatrix} \begin{bmatrix} \alpha_c^2\sigma_c^2+\alpha_x^2\sigma_x^2+\sigma_t^2 & \alpha_c\sigma_c^2 \\ \alpha_c\sigma_c^2 & \sigma_c^2 \end{bmatrix}^{-1} \begin{bmatrix} y-\alpha_0 \\ c \end{bmatrix} \\
&= \frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\left(y-\alpha_0-\alpha_c c\right)
\end{aligned}$$

then:

$$\begin{aligned}
* &= \frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\mathbb{E}_{c|y^a}\left[\left(T\left(c\right)-1\right)\left(y^a-\alpha_0-\alpha_c c\right)|y^a\right] \\
&= \frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\mathbb{E}_{c|y^a}\left[\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right]
\end{aligned}$$

Proof of Theorem 7

$$\begin{aligned}
\frac{\partial}{\partial y^a}\mathbb{E}_{c,z,x|y^a}\left[m|y^a\right] &= \frac{\partial}{\partial y^a}\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\mathbb{E}_{c|y^a}\left[\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right] \\
&= \frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{\partial}{\partial y^a}\int_{-\infty}^{\infty}\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)f\left(c|y^a\right)dc = *
\end{aligned}$$

Note that the conditional density of $c|y^a$ is known:

$$\begin{aligned}
f\left(c|y^a\right) &= \frac{1}{\delta_2}\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right) \\
\delta_1 &= \alpha_c\frac{\sigma_c^2}{\sigma_y^2} \\
\delta_2 &= \frac{\sigma_c}{\sigma_y}\sqrt{\alpha_x^2\sigma_x^2+\sigma_t^2}
\end{aligned}$$

Using this density and the Leibniz rule:

$$\begin{aligned}
* &= \frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{\partial}{\partial y^a}\int\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\frac{1}{\delta_2}\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&= \frac{\sigma_t^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)}\frac{1}{\delta_2}\int\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c\right)\frac{\partial}{\partial y^a}\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)}\frac{1}{\delta_2}\int\left(1-T\left(c\right)\right)\frac{\partial}{\partial y^a}\left[y^a\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)\right]dc \\
&= -\frac{\sigma_t^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)}\frac{\delta_1}{\delta_2^2}\int\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c\right)\phi'\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{1}{\delta_2}\int\left(1-T\left(c\right)\right)\left[\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)-\frac{\delta_1}{\delta_2}y^a\phi'\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)\right]dc \\
&= -\frac{\sigma_t^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)}\frac{\delta_1}{\delta_2^2}\int\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\phi'\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{1}{\delta_2}\int\left(1-T\left(c\right)\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc = *
\end{aligned}$$

Note that the second part is already an expectation. For the first part I use integration by parts:

$$\begin{aligned}
* &= -\left[\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{\delta_1}{\delta_2^2}\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\delta_2\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)\right]_{c=-\infty}^{c=\infty} \\
&\quad +\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{\delta_1}{\delta_2^2}\int\frac{\partial}{\partial c}\left[\left(1-T\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\right]\delta_2\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{1}{\delta_2}\int\left(1-T\left(c\right)\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc = *
\end{aligned}$$

The first term falls out because the exponential rate of convergence of $\phi\left(c\right)$ is faster than linear.

$$\begin{aligned}
* &= \frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{\delta_1}{\delta_2}\int\left[\left(-T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)+\left(1-T\left(c\right)\right)\alpha_c\right]\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{1}{\delta_2}\int\left(1-T\left(c\right)\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&= -\frac{\sigma_t^2\delta_1}{\sigma_t^2+\alpha_x^2\sigma_x^2}\mathbb{E}_{c|y^a}\left[\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right]-\frac{\sigma_t^2\left[1-\alpha_c\delta_1\right]}{\sigma_t^2+\alpha_x^2\sigma_x^2}\mathbb{E}_{c|y^a}\left[\left(1-T\left(c\right)\right)|y^a\right] = *
\end{aligned}$$

Now I substitute in for $\delta_1=\alpha_c\sigma_c^2/\sigma_y^2$. Also not that $\sigma_y^2=\sigma_t^2+\alpha_x^2\sigma_x^2+\alpha_c^2\sigma_c^2$.

$$\begin{aligned}
* &= -\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\frac{\alpha_c\sigma_c^2}{\sigma_y^2}\mathbb{E}_{c|y^a}\left[\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right]-\frac{\sigma_t^2}{\sigma_t^2+\alpha_x^2\sigma_x^2}\left(1-\frac{\alpha_c^2\sigma_c^2}{\sigma_y^2}\right)\mathbb{E}_{c|y^a}\left[\left(1-T\left(c\right)\right)|y^a\right] \\
&= -\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\mathbb{E}_{c|y^a}\left[\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right]-\frac{\sigma_t^2}{\sigma_y^2}\mathbb{E}_{c|y^a}\left[\left(1-T\left(c\right)\right)|y^a\right]
\end{aligned}$$

Proof of Theorem 8

$$\begin{aligned}
\frac{\partial^2}{\partial\left(y^a\right)^2}\mathbb{E}_{c,z,x|y^a}\left[m|y^a\right] &= -\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{\partial}{\partial y^a}\mathbb{E}_{c|y^a}\left[\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right] \\
&\quad -\frac{\sigma_t^2}{\sigma_y^2}\frac{\partial}{\partial y^a}\mathbb{E}_{c|y^a}\left[\left(1-T\left(c\right)\right)|y^a\right] \\
&= -\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{\partial}{\partial y^a}\int\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\frac{1}{\delta_2}\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\sigma_y^2}\frac{\partial}{\partial y^a}\int\left(1-T\left(c\right)\right)\frac{1}{\delta_2}\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc = *
\end{aligned}$$

The Leibniz rule enables me to switch derivation and integration:

$$\begin{aligned}
* &= \frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{\delta_1}{\delta_2^2}\int\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\phi'\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad +\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{1}{\delta_2}\int\left(T'\left(c\right)\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\frac{\sigma_t^2}{\sigma_y^2}\frac{\delta_1}{\delta_2^2}\int\left(1-T\left(c\right)\right)\phi'\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc = *
\end{aligned}$$

Now I use integration by parts for both the first and the third term:

$$\begin{aligned}
* &= \left[\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{\delta_1}{\delta_2^2}\left(T'\left(c\right)\right)\left(\alpha_0+\alpha_c c-y^a\right)\delta_2\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)\right]_{-\infty}^{\infty} \\
&\quad -\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{\delta_1}{\delta_2}\int\left[T''\left(c\right)\left(\alpha_0+\alpha_c c-y^a\right)+T'\left(c\right)\alpha_c\right]\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad +\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\frac{1}{\delta_2}\int\left(T'\left(c\right)\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc \\
&\quad -\left[\frac{\sigma_t^2}{\sigma_y^2}\frac{\delta_1}{\delta_2}\left(1-T\left(c\right)\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)\right]_{-\infty}^{\infty} \\
&\quad -\frac{\sigma_t^2}{\sigma_y^2}\frac{\delta_1}{\delta_2}\int T\left(c\right)\phi\left(\frac{c-\delta_1\left(y^a-\alpha_0\right)}{\delta_2}\right)dc = *
\end{aligned}$$

The two bracketed expression is zero as $\phi(c)$ converges to zero faster than all the other terms. The rest of the terms are in expectation form:

$$\begin{aligned}
* &= -\frac{\sigma_t^2\alpha_c\sigma_c^2\delta_1}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\mathbb{E}_{c|y^a}\left[T''(c)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right] \\
&\quad+\left(\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\left(1-\alpha_c\delta_1\right)-\frac{\sigma_t^2}{\sigma_y^2}\delta_1\right)\mathbb{E}_{c|y^a}\left[T'(c)|y^a\right] \\
&= -\frac{\sigma_t^2\alpha_c^2\sigma_c^4}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^4}\mathbb{E}_{c|y^a}\left[T''(c)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right] \\
&\quad+\left(\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\left(1-\frac{\alpha_c^2\sigma_c^2}{\sigma_y^2}\right)-\frac{\sigma_t^2}{\sigma_y^2}\frac{\alpha_c\sigma_c^2}{\sigma_y^2}\right)\mathbb{E}_{c|y^a}\left[T'(c)|y^a\right] \\
&= -\frac{\sigma_t^2\alpha_c^2\sigma_c^4}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^4}\mathbb{E}_{c|y^a}\left[T''(c)\left(\alpha_0+\alpha_c c-y^a\right)|y^a\right] \\
&\quad+\left(\frac{\sigma_t^2\alpha_c\sigma_c^2}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^2}\left(\frac{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)}{\sigma_y^2}\right)-\frac{\sigma_t^2}{\sigma_y^2}\frac{\alpha_c\sigma_c^2}{\sigma_y^2}\right)\mathbb{E}_{c|y^a}\left[T'(c)|y^a\right]=*
\end{aligned}$$

The second term cancels and thus

$$*=\frac{\sigma_t^2\alpha_c^2\sigma_c^4}{\left(\sigma_t^2+\alpha_x^2\sigma_x^2\right)\sigma_y^4}\mathbb{E}_{c|y^a}\left[T''(c)\left(y^a-\alpha_0-\alpha_c c\right)|y^a\right]$$