Mean Reverting Measurement Error in Survival Expectations*

Péter Hudomiet
University of Michigan

JEL codes: C10, I14, J14

December 30, 2014

Abstract

Recent literature found that subjective survival expectations in the Health and Retirement Study (HRS) appear to be overly optimistic among older respondents compared to life table values and actual survival rates. This paper argues that a significant portion of this bias is due to measurement error as opposed to biases in survival beliefs. HRS asks about the subjective probability of living ~12 more years. The life table 12-year survival probabilities are decreasing with age and they are close to zero among the oldest respondents. As the used probability scale in the HRS (0-100%) is bounded, measurement error is likely to be biased away from the boundaries. The paper shows that the extent of this bias is likely to be large. I use a simple measurement error correction model, which assumes that measurement error is mean-zero in the *logit* of the probabilities. The logit transformation leads to an unbounded scale between minus and plus infinity, and the mean-zero measurement error assumption is more reasonable in this metric. The estimated corrected survival beliefs are much closer to actual survival probabilities than the raw survival reports. I discuss the limitations of the simple measurement error model used in the paper, and I discuss a more promising alternative which will be estimated in the future using to-be-collected data in an experimental module of the 2014 wave of HRS.

*Support from National Institute on Aging program project grant 2-P01-AG026571 is gratefully acknowledged. Please send comments or questions to hudomiet@umich.edu.

1 Introduction

Mortality risk is a key factor for many life decisions, such as deciding when to retire, how much to save, whether to purchase health or life insurance, etc. Mortality risk, however, is not directly observable. One approach might be to use life-table survival probabilities. Life-tables, however, only provide variation in mortality risks by age and gender, and these age and gender groups likely differ in many other aspects beyond their survival chances. Another approach might be to use indicators of actual survival in a later wave of a panel survey. Actual survival, however, mixes up genuine mortality risks and luck. In economic models we would want to use only the risk part and not the luck part. These problems motivated data collection on subjective survival expectations, which provides measures of mortality risk on the individual level.

The HRS pioneered in the collection of subjective expectations. The survival question that I use in this paper comes from the 2000 wave of HRS and it reads as follows: "What is the percent chance that you will live to be [target age] or more?" The target age exceeds the individual's age by 10-15 years: it is 75 years for people younger than 65, and 80, 85, 90, 95, and 100 for individuals in successive five-year age intervals.[1]

Recent literature has shown that the HRS subjective survival probabilities work well in the sense that average values of expectations are close to life tables probabilities, subjective expectations correlate with risk factors in reasonable ways, and they predict economic behavior (Smith et al., 2001; Hurd and McGarry, 2002; Hurd et al., 2004; Hurd et al., 2005; Gan et al., 2005; Perozek, 2008; Sloan et al., 2013; Hudomiet and Willis, 2013). It has been documented in the literature, however, that older respondents report overly optimistic answers to subjective survival probability questions in the HRS (Hurd et al., 2005; Elder, 2013; Hudomiet and Willis, 2013). This is sometimes called the flatness bias, because the subjective survival hazards are too flat, they do not decrease fast enough with age. It is not known why this is the case. There are several non-mutually exclusive interpretations of this evidence. First, it may be that life table probabilities are biased downward because they fail to capture improvements in survival across birth cohorts. Perozek (2008), for example, argues that revisions in

[1] HRS actually asks two questions. The first uses the target age 75 and it is asked from everyone who is younger than 65 year old. The second question uses target ages 80-100 depending on the age of the responders. In this paper I use both question but only the 60-90 age sample, so that everyone answers a question about 10-15 year survival.

life table frequencies in the 1990s can be predicted by survival expectations in the HRS. Second, the beliefs of the elderly may be too optimistic.

In this paper I argue for a third explanation of the flatness bias that there is mean-reverting measurement error in the HRS survival expectations. HRS asks about the subjective probability of living 10-15 more years. The life table 10-15-year survival probabilities are decreasing with age quite substantially above age 60, and they are close to zero among the oldest respondents. As the used probability scale in the HRS is bounded to be between 0% and 100%, measurement error is likely to be biased away from these boundaries. For example, imagine that individuals know their actual survival chances precisely, but they report it with some error in the HRS. If individual A's true survival chance is 50%, it seems a fair assumption that his expected report will be around 50%. If individual B's true survival chance is 2%, however, it seems unlikely that his expected report will about around 2%. As the probability scale is bounded at 0%, measurement error is more likely to be positive on average. Older respondents in the HRS might report overly optimistic survival beliefs because of such mean reverting measurement error. Their true survival expectations are close to the 0% bound, and thus, the average measurement error might push their average responses above their true expectations.

There might be other reasons whey measurement error has a mean reverting property. It is possible that 50%, the middle of the probability scale, serves as anchor in the sense that people who are uncertain about the true probability respond with a weighted average of 50% and their "guess" about the true probability. Such anchoring bias might also follow from the fact that people do not want to spend too much time on any individual survey question, and instead, they just give a reasonable answer, which they feel should be close to the middle of the answer scale.

Whatever the reason is for mean reverting measurement error, it might explain the apparent flatness bias in reported subjective survival expectations. In order to correct for this bias we need to estimate a map between underlying beliefs and survey responses. In an experimental module of the 2014 HRS, we are asking a series of expectation questions that will enable us to do exactly that. Section 5 shall discuss this approach in more detail.

In this paper I use a simpler approach to approximate the extent of mean-reverting measurement error. I use a simple statistical model that assumes that measurement error is mean-zero in the *logit* of the probabilities as opposed to in the level of the probabilities. This assumption is arguably strong and it only serves as a benchmark for approximating the extent of the bias. The logit transformation has the advantage that it is unbounded, it is defined on the entire minus and plus infinity scale. It is more reasonable to assume that measurement error is mean-zero on an unbounded scale. The logit transformation is only one, and quite arbitrary, method to make the scale unbounded, and future research should test its validity. Nevertheless, I shall show that the estimated corrected survival beliefs, using the logit transformation, are much closer to actual survival probabilities than the raw survival reports.

The paper is organized as follows. Section 2 shows the flatness bias in the raw survival expectations. Section 3 lays out several measurement error models, including the simple one that shall be used in the estimation. Section 4 shows the estimation results and Section 5 lays out future work.

2 Flatness bias in survival expectations

Table 1 shows survival expectations and actual survival rates in the 2000 HRS sample by age and gender groups. Actual survival rate is the probability that a person in the 2000 census is still alive in 2012. The actual mortality of HRS respondents is very precisely measured from administrative data (the National Health Index), and it is even available for people who dropped out of the survey in a later wave. Survival expectations are also coming from the same sample in 2000. The subjective expectations ask about the probabilities of living at least 10-15 more years. This is not exactly the same as the 12 year interval used for actual survival, but it should be close to that.

Both the actual and the subjective survival expectations fall with age, but the actual survival probabilities fall faster than the subjective reports. The subjective reports, thus, suffer from a flatness bias. The bias appears to be the strongest in the oldest sample, where the actual survival probabilities are close to zero. For example, among the 85-90 year old male sample, the actual survival chances are around 5%, while the subjective ones are around 29%. For females, the corresponding numbers are 11% (actual) and 31% (subjective). In the next sections I shall investigate whether this flatness bias is due to measurement error or a bias in the beliefs of the elderly.

Table 1: **Survival expectations and actual survival by age and gender**

	Males		Females	
	Actual survival	Survival expectations	Actual survival	Survival expectations
Age 60-64	0.747	0.648	0.835	0.68
	[0.012]**	[0.008]**	[0.011]**	[0.007]**
Age 65-69	0.665	0.564	0.743	0.589
	[0.014]**	[0.009]**	[0.011]**	[0.008]**
Age 70-74	0.509	0.537	0.629	0.541
	[0.015]**	[0.010]**	[0.013]**	[0.009]**
Age 75-79	0.328	0.423	0.43	0.421
	[0.017]**	[0.011]**	[0.014]**	[0.010]**
Age 80-84	0.158	0.377	0.266	0.337
	[0.022]**	[0.015]**	[0.017]**	[0.012]**
Age 85-90	0.049	0.285	0.107	0.307
	[0.031]	[0.021]**	[0.024]**	[0.017]**
N	4594	4594	6223	6223
R squared	0.622	0.764	0.695	0.77

*The table uses data from the 2000 and and 2012 HRS. Actual survival measures the 12-year survival rate of the sample members of the 2000 HRS. Survival expectations are survey reports of surviving 10-15 years in the future in the 2000 HRS.

3 Alternative measurement error models

The goal is to get an unbiased estimate of the average survival beliefs of a particular demographic group, such as age and gender group, using HRS data. I shall use the following notations.

1. p_i^t denotes the true survival probability of individual i, which might differ from his subjective beliefs.

2. p^t denotes the true average survival probability in the group. For example, p^t might be a life table survival probability or the average realized survival chances in the group.

3. p_i^b denotes the belief of individual i about his survival. This variable is unobserved.

4. p^b denotes the average belief in the group.

5. Δ^{b-t} is the average bias in survival beliefs, $\Delta^b = p^b - p^t$.

6. p_i^r denotes the reported survival probability of the individual in the HRS.

7. p^r denotes the average reported survival probability in the group in the HRS.

8. Δ^{r-t} is the average bias in survival reports compared to true survival probabilities, $\Delta^{r-t} = p^r - p^t$.

9. Δ^{r-b} is the average bias in survival reports compared to beliefs, $\Delta^{r-b} = p^r - p^b$.

The goal is to estimate the average survival beliefs in the group, p^b, and the bias in beliefs, Δ^{b-t} .

I assume the following, non-restrictive structure on these probabilities:

$$
\begin{aligned}
f\left(p_i^t\right) &= c + \alpha_i, &(1)\\
f\left(p_i^b\right) &= c + \alpha_i + b + v_i, &(2)\\
f\left(p_i^r\right) &= c + \alpha_i + b + v_i + m_i, &(3)
\end{aligned}
$$

where $f(\cdot)$ is a potentially non-linear function, α_i is an unobserved term in true survival probabilities, b is the average bias in beliefs, v_i is an unobserved term in beliefs, and m_i is measurement error in the HRS reports. I further assume that the unobserved terms are mean-zero and uncorrelated with any other variables. This is a very general parametrization of the problem.

If the $f(\cdot)$ function were known, we could recover the interesting parameters of the model. Using actual survival data we can estimate c using the moment condition

$$c = E\left(f\left(p_i^t\right)\right). \tag{4}$$

Using survival expectations data we can also estimate the bias term, b, using the moment condition

$$b = E\left(f\left(p_i^r\right)\right) - E\left(f\left(p_i^t\right)\right). \tag{5}$$

The problem is, of course, that we do not observe the function $f(\cdot)$. In what follows, I discuss the estimation using different functional forms for the $f(\cdot)$ function.

3.1 Identity function

As a baseline, let us consider the case, when the $f(\cdot)$ is the identity function. The moment conditions in (4) and (5) become:

$$c = E\left(p_i^t\right), \tag{6}$$

$$b = E\left(p_i^r\right) - E\left(p_i^t\right). \tag{7}$$

The bias terms are

$$\Delta^{b-t} = b, \tag{8}$$

$$\Delta^{r-t} = b, \tag{9}$$

$$\Delta^{r-b} = 0. \tag{10}$$

This model implies that survey reports equal true beliefs on average. Consequently, the results in Table 1 would imply that the elderly hold overly optimistic survival beliefs. The differences between the average actual and reported survival probabilities would be unbiased estimates of the biases in beliefs.

3.2 General case

In this section I shall provide formulas for biases in a general, unrestricted case. Given that the function $f(\cdot)$ is non-linear, there is no closed form formula for the biases in beliefs. We can, however, use Taylor approximations for the probabilities and the biases. Let $g(\cdot)$ denote the inverse of $f(\cdot)$, so that $g(f(p)) = p$. The second order Taylor approximation of the true survival probabilities around the sample average $g(c)$ is

$$p_i^t = g(c + \alpha_i) \approx g(c) + g'(c)\,\alpha_i + \frac{g''(c)}{2}\alpha_i^2. \tag{11}$$

By taking expectations we get

$$p^t \approx g(c) + \frac{g''(c)}{2}\sigma_\alpha^2, \tag{12}$$

because $E(\alpha_i) = 0$, by assumption. Similarly we can derive approximate formulas for beliefs and survey reports:

$$
\begin{aligned}
p_i^b &\approx g(c) + g'(c)(b + \alpha_i + v_i) + \frac{g''(c)}{2}(b + \alpha_i + v_i)^2, && (13)\\
p^b &\approx g(c) + g'(c)\, b + \frac{g''(c)}{2}\left(b^2 + \sigma_\alpha^2 + \sigma_v^2\right), && (14)\\
p_i^r &\approx g(c) + g'(c)(b + \alpha_i + v_i + m_i) + \frac{g''(c)}{2}(b + \alpha_i + v_i + m_i)^2, && (15)\\
p^r &\approx g(c) + g'(c)\, b + \frac{g''(c)}{2}\left(b^2 + \sigma_\alpha^2 + \sigma_v^2 + \sigma_m^2\right). && (16)
\end{aligned}
$$

The bias terms are then

$$
\begin{aligned}
\Delta^{b-t} &\approx g'(c)\, b + \frac{g''(c)}{2}\left(b^2 + \sigma_v^2\right), && (17)\\
\Delta^{r-t} &\approx g'(c)\, b + \frac{g''(c)}{2}\left(b^2 + \sigma_v^2 + \sigma_m^2\right), && (18)\\
\Delta^{r-b} &\approx \frac{g''(c)}{2}\sigma_m^2. && (19)
\end{aligned}
$$

Compared to the linear case above, average survey reports differ from average beliefs whenever the $g(\cdot)$ function is non-linear. The bias in survey reports is increasing in the variance of the measurement error. As I shall show below, in reasonable cases the bias has a mean-reverting property.

Unfortunately, however, it is not possible to estimate these bias terms using the available HRS data. For that, we would need to identify the variance of the measurement error σ_m^2 and the variance in belief dispersion, σ_v^2.

3.3 Logit model

In the estimation below, I am going to use the logit transformation for $f(\cdot)$. That is, $f(p) = \ln \frac{p}{1-p}$. The inverse of the logit function and its derivatives are

$$
g(x) = \frac{exp(x)}{1 + exp(x)}, \qquad (20)
$$

$$g'(x) = \frac{exp(x)}{(1+exp(x))^2}, \tag{21}$$

$$g''(x) = \frac{exp(x)(1-exp(x))}{(1+exp(x))^3}, \tag{22}$$

By substituting these equations into the general formulas in the previous section, we get estimators for average probabilities and the different bias terms. For example, the bias terms are

$$\Delta^{b-t} \approx \frac{exp(c)}{(1+exp(c))^2}b + \frac{1}{2}\frac{exp(c)(1-exp(c))}{(1+exp(c))^3}\left(b^2+\sigma_v^2\right), \tag{23}$$

$$\Delta^{r-t} \approx \frac{exp(c)}{(1+exp(c))^2}b + \frac{1}{2}\frac{exp(c)(1-exp(c))}{(1+exp(c))^3}\left(b^2+\sigma_v^2+\sigma_m^2\right), \tag{24}$$

$$\Delta^{r-b} \approx \frac{1}{2}\frac{exp(c)(1-exp(c))}{(1+exp(c))^3}\sigma_m^2. \tag{25}$$

Equation (25) shows that measurement error has a mean reverting property. When $c > 0$, that is, average survival probabilities are above 50%, the measurement error is negative, because $1-exp(c) < 0$. Similarly, when the average survival probability is below 50%, the measurement error is positive.

As before, however, we still cannot identify the biases without measures of σ_m^2 and σ_v^2.

3.4 Simplified logit model

In this section I consider a simplified model, in which the coefficients are identified. Let us assume that the probabilities have the following structure:

$$f\left(p_i^t\right) = c, \tag{26}$$

$$f\left(p_i^b\right) = c+b, \tag{27}$$

$$f\left(p_i^r\right) = c+b+m_i. \tag{28}$$

The only difference between (1)-(3) and (26)-(28) is that the unobserved heterogeneity terms α_i and v_i are shut down. The $f(\cdot)$ function is still assumed to be a logit. Under these conditions the average probabilities are

$$\begin{aligned}
p^t &= g(c), && (29)\\
p^b &= g(c+b), && (30)\\
p^r &\approx g(c) + g'(c)b + \frac{g''(c)}{2}\left(b^2 + \sigma_m^2\right), && (31)\\
&\approx g(c+b) + \frac{g''(c+b)}{2}\sigma_m^2. && (32)
\end{aligned}$$

And the bias terms are

$$\begin{aligned}
\Delta^{b-t} &= \frac{exp\,(c+b)}{1+exp(c+b)} - \frac{exp\,(c+b)}{1+exp(c+b)}, && (33)\\
\Delta^{r-t} &= \frac{exp\,(c)}{(1+exp(c))^2}b + \frac{1}{2}\frac{exp\,(c)\,(1-exp\,(c))}{(1+exp(c))^3}\left(b^2+\sigma_m^2\right), && (34)\\
\Delta^{r-b} &\approx \frac{1}{2}\frac{exp\,(c+b)\,(1-exp\,(c+b))}{(1+exp(c+b))^3}\sigma_m^2. && (35)
\end{aligned}$$

In this model one can estimate all parameters. The variance of m_i can be estimated from the residual variance in (28).

4 Estimation

In this section I am estimating the simplified logit model in Section 3.4 for age and gender groups. The results are in Table 2 (males) and Table 3 (females). The first two columns of the tables are just repeating the actual survival rate and raw expectations from Table 1. Columns 3 and 4 correct for measurement error in the following way. First, I fit the model (28). After fitting the model, I compute average beliefs as the average of $p_i^b = g(c_i + b_i)$ in the sample. The coefficients c_i and b_i are indexed by i because I allow them to vary by demographic groups. As discussed in the previous section, I do not allow unobserved heterogeneity in these variables, so the model in column 4, which allows for more heterogeneity, should be less biased. In column 3 I only include age dummies, and column 4 augments the model with a large number of predictors of survival (education, race, ethnicity, subjective health, smoking and drinking behavior, parents' mortality). The models are estimated by maximum likelihood, and the average survival beliefs are recovered using the delta method averaged within age and gender groups.

Table 2: Imputed survival beliefs and actual survival by age, males

	Actual	Raw expectations	Imputed beliefs	
	[1]	[2]	[3]	[4]
Age 60-64	0.747	0.648	0.735	0.713
	[0.012]**	[0.008]**	[0.013]**	[0.011]**
Age 65-69	0.665	0.564	0.612	0.603
	[0.014]**	[0.009]**	[0.017]**	[0.014]**
Age 70-74	0.509	0.537	0.571	0.566
	[0.015]**	[0.010]**	[0.020]**	[0.017]**
Age 75-79	0.328	0.423	0.358	0.374
	[0.017]**	[0.011]**	[0.020]**	[0.018]**
Age 80-84	0.158	0.377	0.297	0.319
	[0.022]**	[0.015]**	[0.024]**	[0.022]**
Age 85-90	0.049	0.285	0.15	0.171
	[0.031]	[0.021]**	[0.021]**	[0.021]**
N	4594	4594	4594	4594

*The table uses data from the 2000 and and 2012 HRS. Actual survival measures the 12-year survival rate of the sample members of the 2000 HRS. Raw expectations are survey reports of surviving 10-15 years in the future in the 2000 HRS. Imputed beliefs correct the survey reports for measurement error using the model in 3.4. Model 3 uses no covariates other than age group dummies. Model 4 includes other covariates as well: age in years, race (black or other), ethnicity (Hispanic), years of education, subjective health (excellent-very good; good; fair-poor), smoking dummies (smokes now, smoked ever), drinking status (ever drinks, amount of drinking per week on average), whether parents are alive (mom and dad separately).

Table 3: Imputed survival beliefs and actual survival by age, females

	Actual	Raw expectations	Imputed beliefs	
	[1]	[2]	[3]	[4]
Age 60-64	0.835	0.68	0.776	0.753
	[0.011]**	[0.007]**	[0.010]**	[0.009]**
Age 65-69	0.743	0.589	0.648	0.636
	[0.011]**	[0.008]**	[0.014]**	[0.012]**
Age 70-74	0.629	0.541	0.575	0.571
	[0.013]**	[0.009]**	[0.018]**	[0.015]**
Age 75-79	0.43	0.421	0.357	0.373
	[0.014]**	[0.010]**	[0.018]**	[0.016]**
Age 80-84	0.266	0.337	0.216	0.237
	[0.017]**	[0.012]**	[0.015]**	[0.015]**
Age 85-90	0.107	0.307	0.173	0.193
	[0.024]**	[0.017]**	[0.019]**	[0.018]**
N	6223	6223	6223	6223

*The table uses data from the 2000 and and 2012 HRS. Actual survival measures the 12-year survival rate of the sample members of the 2000 HRS. Raw expectations are survey reports of surviving 10-15 years in the future in the 2000 HRS. Imputed beliefs correct the survey reports for measurement error using the model in 3.4. Model 3 uses no covariates other than age group dummies. Model 4 includes other covariates as well: age in years, race (black or other), ethnicity (Hispanic), years of education, subjective health (excellent-very good; good; fair-poor), smoking dummies (smokes now, smoked ever), drinking status (ever drinks, amount of drinking per week on average), whether parents are alive (mom and dad separately).

As we can see, imputed beliefs fall with age much faster than survey reports. That is, after correcting for measurement error, beliefs appear to be less flat. The richer model in column 4 and the less rich model in column 3 are similar. In the oldest sample, however, beliefs are still upward biased.

5 Future work using the HRS experimental modules

The results in the previous section are suggestive that mean reverting measurement in the survival probabilities goes a long way in explaining the flatness bias. The used model, however, is based on many untestable assumptions. It would be valuable to collect and make use of data that enables us to estimate more general models.

Questions in an HRS experimental module in 2014 shall enable us to estimate better models of survey response. The format of the questions in the module is "What is the percent chance that you will live at least X more years?" The fills "X" are age and gender dependent, and they are chosen in each group so that they correspond to similar life table probabilities on average. Each person is asked three questions: one in which the life table probability is around 60 percent, one where it is around 30 percent and one where it is around 15 percent. Having three questions can help identify the response model more precisely. For example, mean-reverting measurement error should have a stronger effect on the third question, where the average answer is around 15 percent compared to the first question. By jointly using the three questions one can derive the extent of mean-reversion in measurement error. The second advantage of having three questions is that the variance of measurement error and belief dispersion can be identified as well. One reasonable assumption one might use is that measurement error is the part of the unobserved heterogeneity in answers that is uncorrelated across questions, and the dispersion is belief is the part that is shared across questions.

References

Elder, T. (2013, April). The Predictive Validity of Subjective Mortality Expectations: Evidence From the Health and Retirement Study. *Demography 50*(2), 569–589.

Gan, L., M. D. Hurd, and D. L. McFadden (2005, October). Individual Subjective Survival Curves. In *Analyses in the Economics of Aging*, NBER Chapters, pp. 377–412. National Bureau of Economic Research, Inc.

Hudomiet, P. and R. J. Willis (2013). Estimating second order probability beliefs from subjective survival data. *Decision Analysis 10*(2), 152–170.

Hurd, M. D. and K. McGarry (2002, October). The Predictive Validity of Subjective Probabilities of Survival. *Economic Journal 112*(482), 966–985.

Hurd, M. D., S. Rohwedder, and J. Winter (2005). Subjective probabilities of survival: An international comparison. Unpublished manuscript, RAND, Santa Monica, CA.

Hurd, M. D., J. P. Smith, and J. M. Zissimopoulos (2004). The effects of subjective survival on retirement and Social Security claiming. *Journal of Applied Econometrics 19*(6), 761–775.

Perozek, M. (2008, February). Using subjective expectations to forecast longevity: do survey respondents know something we do not know? *Demography 45*(1), 95–113.

Sloan, F., L. Eldred, T. Guo, and Y. Xu (2013, August). Are people overoptimistic about the effects of heavy drinking? *Journal of Risk and Uncertainty 47*(1), 93–127.

Smith, V. K., D. H. Taylor, and F. A. Sloan (2001, September). Longevity Expectations and Death: Can People Predict Their Own Demise? *American Economic Review 91*(4), 1126–1134.