Heterogeneity in Expectations, Risk Tolerance, and Household Stock Shares

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The source of heterogeneity in portfolio choices is an important question in household finance (Campbell, 2006). Theories, such as consumption CAPM, predict that the share of risky assets should be positively related to their expected returns, negatively related to their risk, and positively related to investors' risk tolerance. Heterogeneity in preferences and beliefs are therefore natural candidates for explaining heterogeneity in household portfolios.

The existing literature addresses various aspects of heterogeneity. Differences in experience can cause different portfolio choices of households (Malmendier and Nagel, 2011; Seru, Shumway and Stoffman, 2010) as well as professional investors (Vissing-Jorgensen, 2003; Greenwood and Nagel, 2009). These effects may operate through preferences and beliefs as experience can influence both.[1] The association of portfolio allocation with wealth, and individual heterogeneity in that association, may also be driven, at least in part, by expectations and preferences (Calvet and Sodini, 2014). The recent literature has established the role of risk preference and beliefs about future returns in the stock share of household portfolios, although most papers examined stock ownership, that is, the extensive margin, because there are frequently so many households with no stock holdings in the data. The focus on the extensive margin makes quantitative comparisons against benchmark portfolio choice models difficult because their predictions largely concern the magnitude of the intensive margin response of

[1] See, for example, Vissing-Jorgensen (2003), Glaser and Weber (2005), Hurd, van Rooij and Winter (2011), Hudomiet, Kezdi and Willis (2011), Amromin and Sharpe (2012), Hoffman, Post and Pennings (2013), and Guiso, Sapienza and Zingales (2014). Heterogeneity in expectations is also important in other contexts, for example, for the housing market (see Piazzesi and Schneider, 2009) and for inflation (see Malmendier and Nagel, forthcoming; Armantier et al., 2013; Bruine de Bruin et al., 2011).

portfolio shares to preferences and beliefs.[2] Indeed, explaining heterogeneity at the intensive margin, that is, the share of stocks in the portfolio of stock market participants, has remained rather elusive (Brunnermeier and Nagel, 2008). An important reason behind the scarcity of empirical results is the lack of appropriate data. Good data on portfolio composition are needed for a large enough sample of stockholding households, complemented with appropriate measures of preferences and beliefs.

This paper takes a comprehensive approach to examining heterogeneity in portfolio choice by using a distinctive data set created by the Vanguard Research Initiative (VRI) that combines administrative account data and survey responses for a large sample of Vanguard account holders. The VRI has multiple features that make it especially well-suited for examination of heterogeneity in stock holdings.

First, it is a large sample of stock holders. Moreover, despite being drawn from the account holders of a single company, the characteristics of the sample are broadly representative of the targeted population of households with non-negligible financial assets. Hence, unlike

[2] In papers that examine the extensive margin, Barsky, Juster, Kimball and Shapiro (1997), Dohmen, Falk, Huffman and Sunde (2010), Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner (2011) and Guiso, Sapienza and Zingales (2014) show that more risk tolerant individuals are more likely to hold stocks, while Dominitz and Manski (2007) and Hurd, Rooij and Winter (2011) show that individuals with higher levels of stock market expectations and lower perceived risk are more likely to hold stocks. Kimball, Sahm and Shapiro (2008) model the intensive margin. Kezdi and Willis (2011) and Dimmock, Kouwenberg, Mitchell and Peijnenburg (2013) combine the extensive and intensive margins in Tobit-type models and establish associations with risk tolerance, expectations and ambiguity aversion, respectively. Vissing-Jorgensen (2003) and Amromin and Sharpe (2012) show that expectations are related to the share of stocks among stockholders but they do not consider risk tolerance. Weber, Weber and Nosic (2013) show that individual measures of risk tolerance and expectations predict the share of stocks respondents invest in a hypothetical financial portfolio. Hoffmann, Post and Pennings (2013) and Merkle and Weber (2011) analyze the role of expectations and risk tolerance in trading behavior of individual investors rather than the share of stocks in household portfolios.

most studies that focus on the extensive margin for stock holdings, this sample will allow for meaningful inferences about the intensive margin of portfolio choice.

Second, the VRI survey includes batteries of questions that we purposely designed to produce estimates of preference and belief parameters that should help to explain the cross-sectional distribution of portfolio choices. These survey questions yield *quantitative* estimates of individual-level moments of subjective returns distribution and of individual-level values of preference parameters. These estimates can then be related to portfolio decisions in ways that are quantitatively interpretable relative to benchmark economic models.

Third, the design of the VRI allows careful consideration of response errors along a variety of dimensions. These include errors in measuring stock shares in both survey and administrative data and errors in eliciting preferences and expectations from survey responses.

These features—a large, broadly representative sample of stockholders together with quantitative measurements of the potential sources of heterogeneity in stockholding—make the VRI a unique platform for understanding why different households make different portfolio choices.

Section I describes the VRI sample and the measurements of stock share. It addresses the relationship between Vanguard assets and households' overall assets. It also compares administrative and survey measures of portfolio shares. Section II describes how we measure preferences and beliefs. To get individual-specific estimates of preference parameters, we use a modification of the Barsky, Juster, Kimball, and Shapiro (1997) approach of eliciting risk tolerance from hypothetical gambles over permanent income. To get individual-specific estimates of the moments of the perceived distribution of returns, we use both the Manski (2004) approach of eliciting points in the CDF of perceived returns together with individuals' estimates of expected returns. Survey measures of preferences have considerable external validity (i.e., that preference parameters explain a wide range of behaviors) and internal validity (i.e., test-retest validation and consistence across different measures). See Barsky, Juster, Kimball, and Shapiro (1997), Kimball, Sahm, and Shapiro (2008), Dohmen, Falk, Huffman and Sunde (2010), Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner (2011), and Josef, Richter, Samanez-Larkin, Wagner, Hertwig, and Mata (2016) for evidence both of external and internal validity. Recent evidence suggests survey measures of risk preferences show more stability than measures based on small-stakes lottery experiments (Lönnqvist, Verkasalo, Walkowitz and Wichardt, 2015). Similarly, probabilistic measures of expectations have predictive validity (Hurd, 2009; Kezdi and Willis, 2009).

Like many survey measures, preference and expectations are subject to response error. This paper uses a unified procedure accounting for response error to produce unbiased estimates of the subjective variables for both preferences and beliefs. Section III combines these estimates to explain the cross-section of stock shares. We find that the stock share is positively related to the individuals' perceived expected stock returns, is negatively related to their perceived standard deviation of the returns, and is positively related to their risk tolerance. These relationships are economically and statistically significant, they are robust across many specifications, and they are substantially less attenuated than corresponding estimates that do not take care of measurement error in the survey answers. The relative magnitude of the importance of expected returns, standard deviation of returns, and risk tolerance for explaining portfolio shares is quite close to the predictions of benchmark theory, though the absolute magnitudes are much smaller than theory would predict. Additional results suggest that the selected nature of our sample is unlikely to explain this attenuation.

Hence, though the results show that it is possible to use survey responses about economically-relevant subjective variables to explain meaningful features of stock holding, the actual distribution of stockholding varies less with the subjective variables than theory would predict given the measured heterogeneity in subjective variables. We call the finding that portfolio shares have a damped response to preferences and beliefs the "attenuation puzzle." The paper uses two benchmark models of portfolio choice to assess the relationship between *quantitative* measures of preferences and beliefs—the classic Merton model and a richer lifecycle model that makes more realistic assumptions about the environment for portfolio choice. The paper's contributions to the measurement of preferences and beliefs address head-on potential explanations for the attenuation puzzle based on the measurement and modeling of preferences and beliefs:

- First, in contrast to measures of risk tolerance and expected returns based on loose or vague attitudinal scales, this paper presents quantitative estimates of the preference and belief parameters that theory mandates should determine portfolio choice.
- Second, this paper uses a statistical approach where the estimated individual-specific preference and belief parameters are by construction uncorrelated with the measurement errors that arise from the response errors that are inherent in eliciting subjective responses from individuals.

Hence, the estimated relationships presented in the paper are not subject to the attenuation biases that arise from having regressors that are only loose proxies for the variables of interest or that are subject to classical errors in variables. Consequently, our findings imply that the attenuation puzzle is a feature of investor behavior that is not well-captured by benchmark models.

I. VRI Data and Stock Share Measurement

A. VRI sample

The Vanguard Research Initiative (VRI) consists of linked survey and administrative data of account holders who have non-negligible financial assets at Vanguard, are at least 55 years old, and use the Internet to access their Vanguard accounts. This last requirement is necessary because the VRI is an Internet survey. The VRI is an individual level survey, but it includes questions about household-level wealth and income as well as questions about spouses' or partners' demographics and labor supply. The survey oversampled older account holders and singles. The VRI draws respondents from two lines of business—individual account holders and employer-sponsored account holders.[3]

We use responses to three VRI surveys, conducted in the fall of 2013, winter of 2014 and summer of 2014.[4] The main focus of the first survey was to inventory income, wealth and portfolio of households as well as to gather basis demographics. The second survey implemented Strategic Survey Questions (SSQs), which ask respondents to make choices under hypothetical situations designed to elicit meaningful preference data. This paper uses the questions about risk preference. The third survey includes the questions about beliefs about returns used for this paper, and also covers a number of issues not related to this paper. 4,730 respondents completed all the three surveys. The item non-response rate of the VRI is

[3] The employer-sponsored are enrolled at Vanguard through 401(k) or similar defined-contribution accounts. While both individual and employer-sponsored account holders are selected via ownership of a Vanguard account, the selection into individual and employer-sponsored accounts is presumably quite different. We will present separate estimates to get a sense of whether selection matters for results.

[4] The plan is to implement the VRI as a panel. These three surveys, however, cover distinctive topics with little longitudinal content. They were broken into three surveys of 40 to 60 minutes for the practical reason of not overwhelming respondents.

remarkably low. Our analysis includes the 4,414 respondents with non-missing observations for all the variables used in the analysis.

The VRI sample frame is based on administrative account data for Vanguard. Having such data to create a sample is an important element of the VRI design. Additionally, administrative data are composed of monthly history of Vanguard assets, with information on types, balances and stock shares of the accounts linked to the survey measures. This paper uses both survey and administrative measures of assets and their composition. The survey measure covers all assets, not just those held at Vanguard. See Ameriks, Caplin, Lee, Shapiro and Tonetti (2014a, 2014b) for a detailed discussion of the design of the VRI including sampling and response rates, and of the VRI's approach to wealth measurement.

Details of the measurement and distributions of stock shares, preference parameters, and stock market expectations will be discussed in the next sections. Here we briefly describe the measurement of wealth and other variables that are used in the analysis: marital status, gender, age, education, earnings, annuity income, expectations about longevity and long-term care use..

The VRI survey measure of wealth is based on a comprehensive account-by-account approach. The survey first asked about types of accounts respondents have (e.g. IRA, checking, money market funds) and the number each type of account held by the respondent or her spouse. For each account they indicated owning, the respondents were asked to provide the balance as well as the share of stock-market assets. When finished with all accounts, respondents were presented a summary table consolidating their responses and were invited to make corrections, if any.

Measuring wealth and stock shares account by account matches the way respondents keep track of their own wealth, and it does not require them to sum balances across accounts to provide total figures for asset categories that are familiar to economists but less so to survey respondents. In contrast, the Health and Retirement Study (HRS) and the Survey of Consumer Finances (SCF)—other leading surveys with state of the art wealth measurement—use account-by-account approaches but only for selected sets of account types. Item non-response in the wealth section of the VRI affects less than 1 percent of the observations.[5]

Table 1. Summary Statistics: VRI, HRS, and SCF

	VRI		HRS	SCF
	Entire sample	Analysis sample	VRI-eligible subsample	
Household-level variables				
Number of households	8,950	4,414	3,684	1,275
Number of stockholding households	8,636	4,323	2,356	1,216
Average financial wealth ($'000)	1,207	1,148	578	970
Average total wealth ($'000)	1,589	1,551	804	1,764
Average stock share among stockholders	0.56	0.56	0.55	0.46
Respondent-level variables				
Married	0.67	0.68	0.70	0.71
Male	0.64	0.65	0.56	0.79
Age	67.8	67.8	64.9	64.1
Less than college degree	0.30	0.26	0.51	0.45
College degree but not more	0.32	0.33	0.23	0.27
Post-college degree	0.38	0.41	0.26	0.28
Retired	0.56	0.60	0.53	0.34

Notes. For the HRS and SCF, the VRI-eligible subsamples are those who are not younger than 55, have access to the Internet at home, and have at least $10,000 in non-transactional accounts. Respondent-level variables for the HRS refer to the financial respondents; for the SCF they refer to the household heads. Variables in the VRI measured in 2013; HRS and SCF are from 2012 and 2013, respectively. Respondent-level variables are {0,1} binary variables except for age.

[5] Summary statistics of the wealth measures are shown in Table A1 in the Appendix. Table A2 in the Appendix shows the summary statistics of the variables we use as controls in our analysis, together with the definition of those variables.

Table 1 compares the VRI sample to the HRS and SCF.[6] The HRS and SCF are nationally representative samples (of those above age 50 in the case of the HRS). Table 1 compares the VRI sample to the subsample of the HRS and SCF after imposing restrictions similar to VRI eligibility: being at least 55 years old, having access to Internet at home, and having at least $10,000 financial wealth. The table shows the number of households, the number of stock holding households, average financial wealth and total wealth, average stock shares, and some demographic characteristics of the individuals responding each survey.

The number of respondents who completed Survey 1 is substantially larger than the VRI-eligible subsample of the HRS and the SCF. The difference in the number of respondent in stock holder households is even larger: while the parallel sample has slightly over 1,000 stock-holding households in the SCF and slightly over 2,000 in the HRS, the entire VRI sample has more than 8,000 stock holders and the sample used in our analysis has more than 4,000.

The demographic composition of the VRI sample is very similar to the parallel subsamples of the HRS and the SCF. Average total wealth and average financial wealth in the VRI are remarkably close to corresponding estimates from the SCF; the HRS estimates are lower. The average stock share in financial wealth among stock holders is very similar in the VRI and the HRS; the SCF estimates are somewhat smaller. VRI respondents are slightly less likely to be married, and they are somewhat older, more educated and more likely to be retired. The differences in marital status, age and retirement are largely due to the fact that the VRI oversampled older individuals and singles. 65 percent of the VRI sample is male, compared to 79 percent in the SCF and 56 percent in the HRS. While the respondent-level compositions are

[6] This comparison with the HRS and the SCF draws on Ameriks, Caplin, Lee, Shapiro and Tonetti (2014a). See this for further details about the VRI sample and wealth measurement.

arbitrary to some degree (account holders in the VRI, financial respondents in the HRS, and household heads in the SCF), the fact men are overrepresented in all samples reflects that they are more likely to own accounts with substantial wealth. The sample used in our analysis is very similar to the initial VRI sample indicating that attrition between the VRI surveys was close to be random.

B. Measuring stock shares

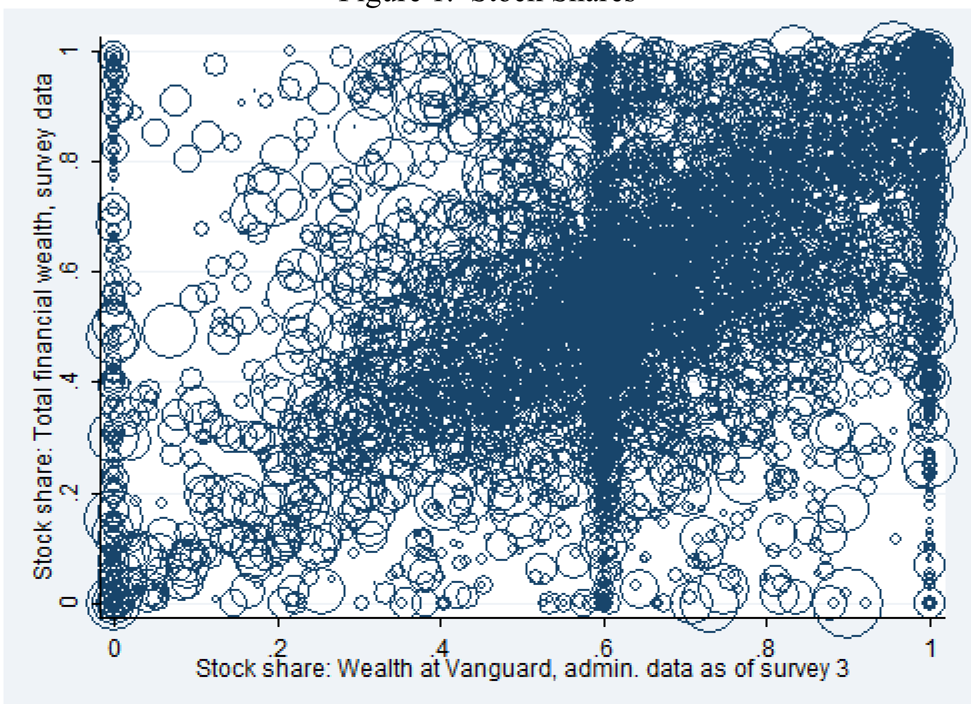
Our analysis focuses on the share of stock market-based assets in total financial wealth.[7] The stock share in financial wealth is the weighted average of the stock shares of the accounts as reported by the respondents. Respondents who did not answer all of the account-by-account stock share questions were asked the overall stock share of their financial portfolio. Ninety-five percent of respondents answered all the account-by-account stock share questions; the distribution of stock share is very similar across the two groups.

Besides the overall stock share we also analyze the stock share in wealth held at Vanguard based on administrative data. The monthly history of accounts in the VRI administrative data breaks down the balance of each account into stock, bond and money market holdings. This break-down is not readily available for all accounts, so we imputed stock share when needed using information on the type of fund the account is invested in (e.g., for an account invested in a balanced fund, we assume 60% of stock share). The administrative stock share measure is available both at around the time when the stock expectation questions are asked and also at the time when the survey measure of household portfolio is obtained (the wealth survey took place in the fall of 2013, while expectations were asked in the summer of

[7] Specifying stock share in financial wealth is standard in the literature. Alternative measures may include housing wealth and human capital wealth in the denominator. We include such wealth items as control variables in the analysis and show that their inclusion leads to very similar results for the parameters of interest.

2014). At the same time, the administrative stock share measure corresponds to the subset of financial wealth held at Vanguard.

Figure 1. Stock Shares



Note: The figure plots the administrative measure of stock share at Vanguard at the time of Survey 3 on the horizontal axis versus the survey measure of stock share overall at the time of Survey 1 on the vertical axis. These are the main dependent variables in the analysis. The size of the marks on the figure is proportional to the Vanguard financial wealth of the respondents.

Figure 1 compares the stock share measures based on the survey and the administrative data that will be the main dependent variables for our analysis. The horizontal axis shows the administrative measure of the stock share in assets held at Vanguard at the time we measured expectations in Survey 3, while the vertical axis shows the Survey 1 measure of stock share in total financial assets.[8] The size of the marks on the figure is proportional to the Vanguard financial wealth of the respondents. The figure shows that many observations are near the 45-

[8] Due to the imputation used for the balanced funds, there is bunching at 60 percent according to the administrative measure.

degree line, so as a practical matter either measure may provide similar inferences for many respondents. At the same time, the two are often different. Indeed, the correlation coefficient is only 0.61.[9] The two measures can be different for three main reasons. First, they are measured at different times, in summer 2014 versus fall 2013. Second, they refer to different sets of assets: Vanguard assets versus all financial assets. Third, they are measured in different ways: using administrative records versus answers to survey questions.

It is important to understand which of these differences matter. Figures 2 through 4 compare alternative stock share measures broken down by the time of measurement, the accounts covered, and the method of measurement (survey or administrative). First, consider the time dimension. The horizontal axis of Figure 2 shows the stock share in assets held at Vanguard at the time of Survey 1 (fall 2013), while the vertical axis shows the same, measured at the time of Survey 3 (summer 2014). Almost all observations are on the 45 degree line, and the correlation is very strong (0.95), implying that the stock share changed little over this time period. As a practical matter, this means that changes in portfolios between administration of the VRI surveys is very small, so that differences in when the various questions were fielded is likely not that important. It is also of substantive interest that portfolio shares are so sticky, something that we see also in much longer intervals of the administrative data.

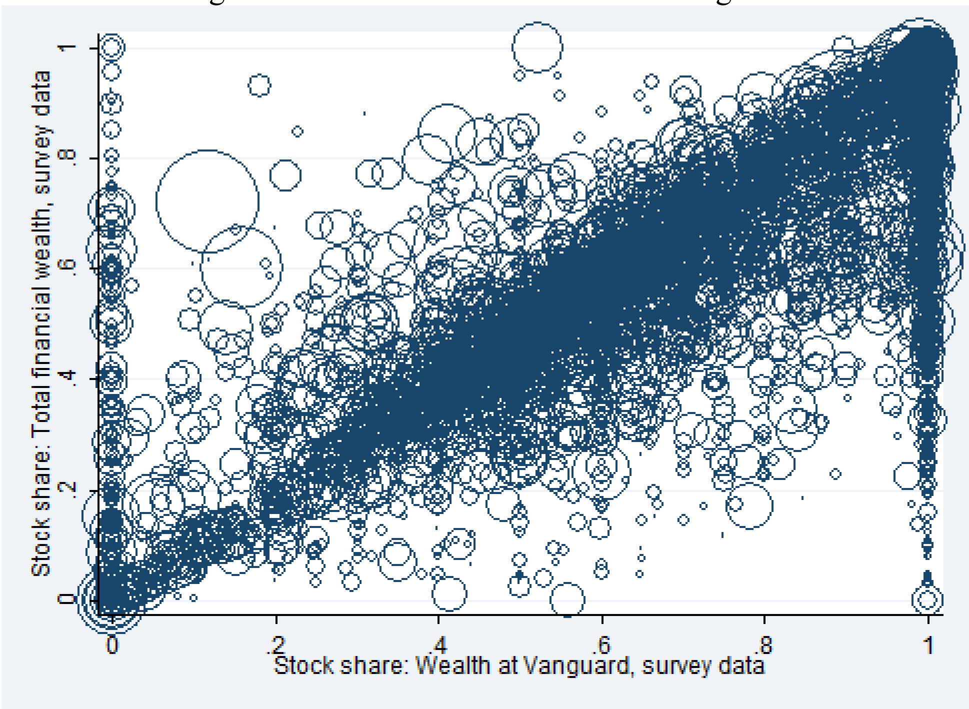
[9] All figures and correlation coefficients are weighted by Vanguard wealth. The unweighted correlation is 0.40, indicating that deviations are somewhat larger if wealth is low.

Figure 2. Stock Shares: Differences in Time

Note: The figure plots the administrative measure of stock share at Vanguard at the time of Survey 3 on the horizontal axis versus the administrative measure of stock share at Vanguard at the time of Survey 1 on the vertical axis. The size of the marks on the figure is proportional to the Vanguard financial wealth of the respondents.

Second, consider the issue of whether or not the assets are held at Vanguard. Figure 3 plots stock share at Vanguard versus overall, both measured as survey responses in Survey 1. There is relatively high correlation in the stock shares (correlation = 0.81), so differences in portfolio shares across providers, though not trivial, is not the main source of the dispersion shown in Figure 1. The difference can be small because account holders have most of their assets at Vanguard, or that they have asset compositions that are similar across Vanguard and non-Vanguard providers.

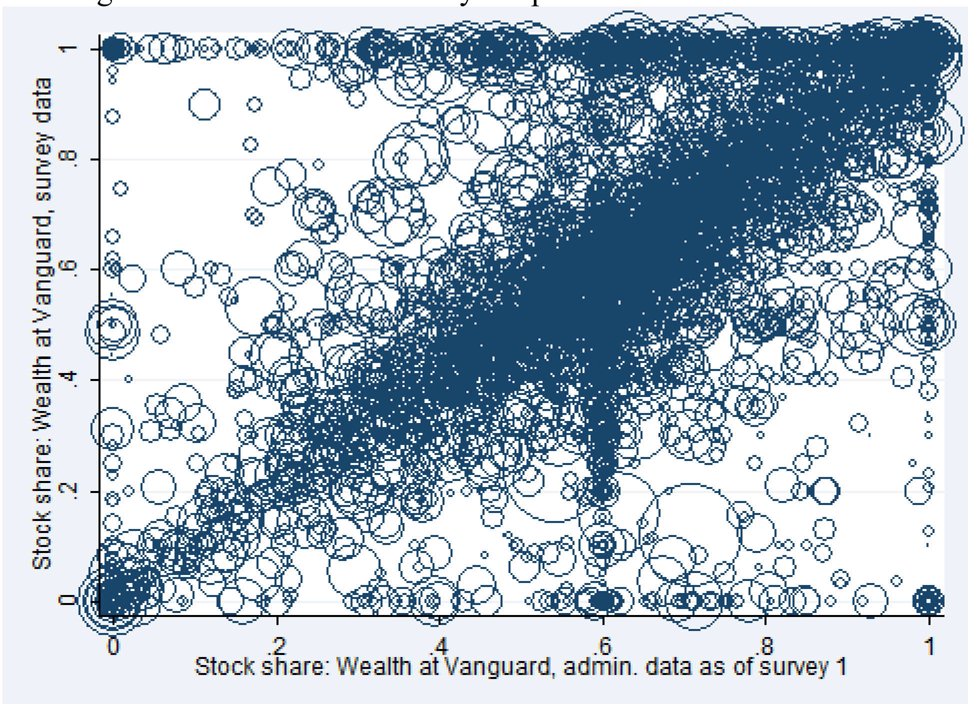
Figure 3. Stock Shares: Total versus Vanguard



Note: The figure plots the survey measure of stock share at Vanguard on the horizontal axis versus the survey measure of stock share overall on the vertical axis. The size of the marks on the figure is proportional to the Vanguard financial wealth of the respondents.

Third, Figure 4 shows the stock share at Vanguard at the time of Survey 1. Administrative data are on the horizontal axis and survey data are on the vertical axis. The dispersion is very much as in Figure 1 (correlation = 0.64). Hence, it turns out that the main source of the dispersion is the deviation between survey and administrative measurements, not difference in timing or difference arising from Vanguard versus overall portfolios.

Figure 4. Stock Shares: Survey Response Versus Administrative



Note: The figure plots the survey measure of stock share at Vanguard on the horizontal axis versus the survey measure of stock share overall on the vertical axis. The size of the marks on the figure is proportional to the Vanguard financial wealth of the respondents.

Several findings of independent interest emerge. First, based on the administrative data, portfolio shares are quite sticky over time. Second, the deviations of survey and administrative measures of portfolios suggest that individuals perceive different stock exposure than they have at any moment. Both these findings present challenges to standard theories of portfolio choice and therefore affect the interpretation of results relating portfolio choices to preferences and beliefs. We return to these issues after presenting the results.

II. Measuring Preferences and Expectations

A. Measuring risk tolerance

Survey 2 of the VRI included Strategic Survey Questions (SSQs) that ask respondents to make choices between hypothetical financial products under hypothetical situations. By specifying hypothetical situations that are independent from their own economic, health and family conditions, these SSQs enable us to better estimate structural preference parameters. This approach to measurement was pioneered by Barsky, Juster, Kimball, and Shapiro (1997) for measuring risk preference in the HRS and Ameriks, Caplin, Laufer, and Van Nieuwerburgh (2011) for measuring preferences surrounding long-term care. The approach is refined and extended in the VRI (see Ameriks, Briggs, Caplin, Shapiro and Tonetti, 2015a, b). In this paper, we use the VRI's risk tolerance questions that pose gambles over consumption. The VRI risk preference questions ask about preference between having a certain level of consumption and a 50-50 chance of having double that level of consumption versus having it fall by x%. It then alters the downside risk x to partition respondents into risk tolerance groups.[10] Table A3 in Appendix A gives the exact wording of the risk tolerance question in the VRI.

The question is asked for two different levels of riskless consumption, \$100K and \$50K per year, and downside risks of 1/10, 1/5, 1/3, 1/2, and 3/4. Table 2 shows the distribution of the answers to the two questions. Most respondents have low tolerance for risk. About half of the respondents chose the first two categories, indicating that they would not accept a risk of more than 20% drop in their consumption to take a chance to double their consumption. Only a small fraction chose the last two categories with a risk of more than a 50% drop. Overall, the distribution is similar to the distribution of the answers to a similar question in the HRS except that the fraction of respondents in the two extreme categories (0-10% and 75-100%) is slightly lower in the VRI (see Kimball, Sahm, and Shapiro 2008 for HRS).

[10] The original HRS question has the same structure, but asks about gambles over life-time income rather than consumption.

The table also shows that more respondents fall into the lower risk categories when riskless consumption is $50,000 instead of $100,000. We will handle this increase relative risk tolerance by positing a utility function with a subsistence level of consumption. Following Barsky, Juster, Kimball, and Shapiro (1997) and Kimball, Sahm and Shapiro (2008), we will use the multiple responses to identify the heterogeneity of the preference parameter and survey response errors (see details below).

Table 2. Risk Tolerance: Distribution of Responses to SSQ

Response category	Downside risk accepted	Downside risk rejected	Percent of answers: riskless consumption $100K	Percent of answers: riskless consumption $50K
1	none	1/10	23	28
2	1/10	1/5	26	34
3	1/5	1/3	26	26
4	1/3	1/2	13	9
5	1/2	3/4	10	3
6	3/4	none	2	1
Total			100	100

Choice between two plans. Plan A guarantees $W consumption next year. Plan B: doubles $W with 50% chance and cuts it by a fraction x with 50% chance. $W=100K or 50K, shown in the two columns; the x values are shown in second and third columns. 4414 observations.

Estimation of a cardinal risk tolerance parameter requires specifying a utility function. We assume that the flow utility function is a generalization of CRRA with a subsistence level of consumption

$$u_i(c) = \frac{(c+\kappa)^{1-1/\gamma_i}}{1-1/\gamma_i}, \qquad (1)$$

where subscript i denotes heterogeneity across individuals, c is consumption, the negative of κ is the subsistence level of consumption, assumed to be the same for all individuals, and γ is the parameter of risk tolerance.

For this utility function, relative risk tolerance (RRT_i) is

$$RRT_i = \gamma_i \frac{c+\kappa}{c} < \gamma_i ,$$

where the risk tolerance parameter γ_i is relative risk tolerance in the $\kappa = 0$ case. See Figure A1 in the Appendix for the relationship of relative risk tolerance and γ as a function of consumption.

To parameterize the heterogeneity of the risk tolerance parameter, we assume that the parameter is distributed lognormally in the population according to

$$\log(\gamma_i) = \overline{\gamma} + u_{\gamma i},\ u_{\gamma i} \sim N(0, \sigma^2_{u\gamma}). \tag{2}$$

We model the measurement error as a log additive term to the parameter, such that

$$\begin{aligned} &\log(\tilde{\gamma}_{ij}) = \log(\gamma_i) + \varepsilon_{\gamma ij} \qquad \text{for } j = 1,2 \\ &\varepsilon_{\gamma ij} \sim N(0, \sigma^2_{\varepsilon\gamma j}) \end{aligned} \tag{3}$$

where γ_i is the true risk tolerance parameter for individual *i*, $\varepsilon_{\gamma ij}$ is measurement error, and $\tilde{\gamma}_{ij}$ is the risk tolerance parameter that provides the basis for individual *i*'s response to the j^{th} question. Thus, in answering question *j* given the level of resource *c* and risk *x* that are associated with the risky gamble, the respondent compares

$$\frac{(c+\kappa)^{1-1/\tilde{\gamma}_{ij}}}{1-1/\tilde{\gamma}_{ij}} \text{ vs. } 0.5\frac{(2c+\kappa)^{1-1/\tilde{\gamma}_{ij}}}{1-1/\tilde{\gamma}_{ij}} + 0.5\frac{((1-x)c+\kappa)^{1-1/\tilde{\gamma}_{ij}}}{1-1/\tilde{\gamma}_{ij}} \tag{4}$$

to determine whether to accept the risky gamble or not. This approach follows Kimball, Sahm, and Shapiro (2008). We carried out the estimation procedure jointly for risk tolerance and stock market expectations, so will defer discussion of estimation until Section II.C below.

B. Measuring beliefs about stock returns

Survey 3 of the VRI asked about beliefs about the one-year return of the U.S. stock market, represented by a stock market index such as the Dow Jones Industrial Average (DJIA).

Respondents had to answer three questions: the expected return on the stock market in the 12 months following the interview (*m*); the percent chance that the stock market will be higher in 12 months following the interview (*p0*) and the percent chance that it will be at least 20% higher (*p20*). The exact wording of the questions is in Table A4 in the Appendix.[11]

Answers to the expected value questions were constrained to be integers. Answers to the percent chance questions were constrained to be 5 point increments between 0 and 15 and between 85 and 100, and they were constrained to be 10 point increments between 15 and 85 (the set $\{0,5,10,15,25,35,45,55,65,75,85,90,95,100\}$). Answers to percent chance questions tend to be rounded to the nearest ten when they are not constrained, with an especially large fraction answering 50 percent (Hurd, 2009). The constraints in the VRI survey forced people to round to other values; in particular, they don't allow for 50 percent answers. Another constraint on the answers ensured that $p20 \leq p0$.[12] No constraints were put on *m* versus *p0* and *p20*.[13]

Table 3 shows the summary statistics of the answers to the questions about the distribution of stock market returns. The survey responses for expected returns (*m*) are distributed around the historical average of 4 to 7 percent depending on sample period, and their dispersion is moderate.[14] In contrast, most answers to the probability questions are lower than the

[11] Bruine de Bruin et al. (2011) and Armantier et al. (2013) examine the reliability of the percent chance questions for inflation as well as how they relate to questions about point expectations of inflation.
[12] Respondents whose initial answer to *p20* violated this constraint are reminded of the constraint by the survey software and asked for a new reply to either *p*0 or *p*20 (or both).
[13] A randomly selected half of the respondents received the *m* question first, followed by *p*0 and *p*20, while the other half received *p*0 and *p*20 first, followed by *m*. The distribution of the responses is different across the two sequences, but those differences do not affect our main results.
[14] Individuals may use different sample windows for inferring expected returns (see Malmendier and Nagel 2011). The table shows some different windows for realized returns. Average returns

historical probabilities, and they have substantial heterogeneity. A non-negligible fraction of the respondents gave a positive number to the expected return question (*m*) and a less than 50 percent chance answer to the probability of a positive return (*p0*). Taken together these answer patterns are consistent with many individuals implicitly applying a positive threshold when they answer the *p0* question (by thinking that the stock market goes up only if it goes up by at least some positive amount).[15]

Table 3. Stock Market Returns: Survey Responses versus Historical Statistics

	Survey answers				Historical statistics			
	Mean	25th pctile	Median	75th pctile	1959-2014	1995-2014	1995-2009	2010-2014
m	0.06	0.04	0.06	0.10	0.04	0.07	0.05	0.13
*p*0	0.51	0.25	0.55	0.75	0.58	0.65	0.53	1.00
*p*20	0.15	0.05	0.15	0.25	0.18	0.25	0.27	0.20

Notes. *m* is expected one-year ahead returns of the stock market index DJIA; *p0* is the probability that the DJIA would be higher a year from the date of the interview; *p20* is the probability that it would be higher by at least 20%. Historical statistics computed from yearly relative returns of the Dow Jones Industrial Average (year on year changes divided by base year value, first days of July in each year), deflated using the PCE chain price index (available beginning in 1959). Historical average values shown for *m*; the fraction of years when positive or greater than 0.2 are shown for *p0* and *p20*. 4414 observations.

In order to use our data more efficiently and in a way that is more informative from a theoretical point of view we map the three survey responses, *m*, *p0*, and *p20* into a perceived returns distribution. The procedure closely parallels that for the risk tolerance questions: the survey responses are based on individual beliefs drawn from normal distribution plus survey response error. We assume that individual *i* believes that yearly returns follow a lognormal

are quite variable owing to the well-known problem of estimating the expected return on the market.

[15] Glaser, Langer, Reynders and Weber (2007) document a similar pattern when they compare stock market expectations elicited in terms of returns versus prices. They label the phenomenon as "framing effect," and our explanation can be viewed as a source of such a framing effect. Note that, although skewed returns could explain the phenomenon we observe, it is an unlikely explanation. The combination $m_i>0$ and $p0_i<0.5$ would correspond to long positive tails, implying mean above the median and infrequent large gains. This skewedness is the opposite of what one would expect from a "black swan" theory of infrequent stock market crashes.

distribution with individual-specific mean and standard deviation of log stock returns of μ_i and σ_i. Similar to how we handle the cross-sectional distribution of the risk tolerance parameter, these parameters are drawn across individuals as

$$\begin{matrix} \mu_i = \bar{\mu} + u_{\mu i} \\ \sigma_i = \bar{\sigma} + u_{\sigma i} \end{matrix}, \quad \begin{pmatrix} u_{\mu i} \\ u_{\sigma i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u\mu}^2 & \rho_{\mu\sigma}\sigma_{u\mu}\sigma_{u\sigma} \\ . & \sigma_{u\sigma}^2 \end{pmatrix} \right). \tag{5}$$

Individuals answer the survey questions *m*, *p*0 and *p*20 based on their beliefs, but their answers contain survey noise, that is, measurement error specific to the survey situation. Using the structure of the survey questions on expected returns and the two points of the probability distribution, applying the assumption of lognormal returns, and adding survey response error yields

$$\tilde{m}_i = \mu_i + \varepsilon_{mi}, \; \varepsilon_{mi} \sim N(0, \sigma_{\varepsilon m}^2) \tag{6}$$

$$\tilde{p}_{0i} = \Phi(\frac{\mu_i}{\sigma_i} + \varepsilon_{0i}), \; \varepsilon_{0i} \sim N(\psi, \sigma_{\varepsilon p}^2) \tag{7}$$

$$\tilde{p}_{20i} = \Phi(\frac{\mu_i - 0.2}{\sigma_i} + \varepsilon_{20i}), \; \varepsilon_{20i} \sim N(0, \sigma_{\varepsilon p}^2) \tag{8}$$

where $\tilde{m}_i$, $\tilde{p}_{0i}$, and $\tilde{p}_{20i}$ are the error-ridden variables that determine survey responses. Survey error is assumed to be independent across the three answers, with mean zero except for *p*0 where its mean is ψ , which allows for the documented differences between *m* and *p*0. An interpretation of ψ is that, on average, respondents answer the question about positive returns (*p*0) as if they had some positive threshold in mind instead of zero ($\Phi(-\psi/\sigma_i)$, $\psi < 0$). The variables $\tilde{m}_i$, $\tilde{p}_{0i}$, and $\tilde{p}_{20i}$ are before rounding. Recall that the VRI probability scale is for rounded responses. Similarly, as discussed above, the risk tolerance questions yield discrete responses. The next section discusses how our estimation procedure handles this issue.

C. Joint estimation of heterogeneity in stock market expectations and risk tolerance

Given the models of heterogeneity in preferences and beliefs (equations (2) and (5)), the structural interpretation of the survey questions together with the additive survey response errors ((3), (4), (6), (7) and (8)), we can now move to estimation of the model. The parameters to be estimated are $\overline{\gamma}, \overline{\mu}, \overline{\sigma}, \sigma^2_{u\gamma}, \sigma^2_{u\mu}, \sigma^2_{u\sigma}, \rho_{\mu\sigma}, \psi, \sigma^2_{\varepsilon\gamma 1}, \sigma^2_{\varepsilon\gamma 2}, \sigma^2_{\varepsilon m}, \sigma^2_{\varepsilon p}$. We allow for $\overline{\gamma}$, $\overline{\mu}$, $\overline{\sigma}$, and ψ to vary with covariates. Additionally, we allow the beliefs about returns to depend on risk preference, so the covariates of $\overline{\mu}$ and $\overline{\sigma}$ include the latent γ_i. The estimation method is maximum likelihood. It allows for interval responses to the risk tolerance question and the returns questions. Appendix B shows the likelihood function.

Table 4. Distribution of Preferences and Beliefs

		Mean	Standard deviation	25th pctile	Median	75th pctile
Preferences						
Risk tolerance parameter	γ_i	0.41	0.33	0.20	0.32	0.50
Subsistence consumption	$-\kappa$	17,000				
Beliefs						
Mean of return	μ_i	0.06	0.06	0.02	0.06	0.10
Standard deviation of return	σ_i	0.12	0.03	0.10	0.12	0.14

Notes. Statistics are calculated from the estimated parameters in Table A5; see the notes to Table A5 for more detail.

Table 4 shows key estimated statistics of the distribution of preferences and beliefs based on the estimated statistical model of preferences, beliefs, and response error. Table A5 in the appendix shows the estimates of the underlying parameters of the model.[16] The subsistence level of consumption (−κ) is estimated to be $17,000. The negative value of κ generates decreasing

[16] The summary statistics in Table 4 are from estimates without covariates. Appendix Table A6 reports the estimates of the statistical model with covariates.

relative risk aversion as in the basic/luxury good model of Wachter and Yogo (2010).[17] The estimated mean of the risk tolerance parameter (γ) implies low risk tolerance on average. A respondent with the mean level of γ and κ has relative risk tolerance 0.34 (relative risk aversion 2.9) when the consumption level is $100,000. In terms of the SSQ question, she would be indifferent between a fixed consumption of $100,000 and the 50-50 gamble of doubling that consumption and losing 20 percent. There is a considerable heterogeneity in risk tolerance. At the 25th percentile of risk tolerance parameter, the point of indifference is the downside risk of losing 13 percent; at the 75th percentile the point of indifference is the downside risk of losing 29 percent. These numbers indicate higher levels of risk tolerance than in a representative sample of Americans older than 50 years of age. Kimball, Sahm and Shapiro (2008) estimate the corresponding risk tolerance percentiles (25th, 50th and 75th) to imply indifference to 7, 12 and 20 percent of downside risk, respectively.

Beliefs about mean stock returns are in line with historical mean returns, on average. Beliefs about standard deviation are slightly lower than the historical value of 0.16. Heterogeneity in perceived mean returns (μ) is substantial, with the lowest 25 percent believing expected returns to be 2 percent or less and the top 25 percent believing 12 percent or more. At the same time, estimated heterogeneity in the perceived standard deviation of stock returns (σ) is small, perhaps because it is easier for people to estimate the second moment of the returns distribution than the first moment, as pointed out by Merton (1980).[18,19]

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[17] The design of the SSQ does not allow heterogeneity in κ to be readily identified, although it tightly identifies its mean.

[18] According to our estimates heterogeneity in preferences and beliefs are weakly related. More risk tolerant respondents believe that stock returns are slightly higher, but we don't find association of risk tolerance and beliefs about the standard deviation of returns. Beliefs about the mean and the standard deviation of returns are weakly positively correlated.

Based on the estimated distribution summarized in Table 4, 17 percent of the population expects negative stock returns. As we will see, this part of the population holds less stock than on average, but still has substantial stock market exposure. Symmetrically, 17 percent expect returns to be larger than 12 percent, rates of return that should make people hold the vast majority of their wealth in stocks given the distribution of risk and risk preferences. Though this part of the population holds more stock than on average, very high stock shares are uncommon. Taken together, these facts suggest that expectations translate into stock shares in an attenuated fashion, a finding that our analysis will verify in the next section.

The Table 4 results take into account substantial estimated survey noise. Again, the parameters of the survey noise distributions are presented in Appendix Table A5. To understand the magnitude of noise, consider the differences in terms of the survey responses of individuals with the estimated averages of latent preferences and beliefs, one without measurement error and one with a positive standard deviation unit shock of measurement error. A one standard deviation unit measurement error in the first risk tolerance SSQ would make the survey response imply a point of indifference of a 38% drop of consumption instead of the 20% implied by an error-free answer. A one standard deviation unit measurement error in the second risk tolerance SSQ would make the response imply an indifference point of 27% instead of 17%. One standard deviation unit measurement error in the response to the expected stock returns question would result in a

[19] Preferences and beliefs are significantly related to observable right hand side variables in our sample (Table A5 in the Appendix). However, when interpreting these associations, one has to keep in mind that the VRI sample is selected on wealth and stock ownership. For example, sample selection may explain the negative correlation of wealth and stock market expectations. Almost all households in the VRI sample have nonzero stockholding. With fixed costs of stock market participation wealth should matter at the extensive margin on top of expectations. As a result, we expect wealthier stockholders to have lower expected returns than less wealthy stockholders.

response of 14% instead of 6%; one standard deviation unit measurement error in the stock market probability answers would change *p0* responses to 67% from 48% and *p20* responses to 25% from 12%. The estimated bias of the measurement error in the *p0* response (ψ) suggests that, on average, people think of positive gains only when they exceed 4 percent when answering the $p0$ question.[20]

D. Estimating individual-specific cardinal proxies of risk tolerance and beliefs

In the previous sections, we show how to separately identify the true heterogeneity in preferences and beliefs and the survey response errors in the survey measures of them. In this subsection, we explain how we construct the individual-specific belief and preference parameters based on those estimates that are immune from the standard effects of using generated regressors.

1. *Constructing individual-specific preference and belief parameters*

Using the estimation results we calculate individual-specific proxy variables $\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\gamma}_i$. These proxies are the expected values of the corresponding latent variables: the individual-specific expected value and standard deviation of the distribution of stock market returns perceived by the individual (μ_i, σ_i), and the individual-specific latent parameter of risk tolerance (γ_i). They are expected values conditional on the individual's responses to the survey questions on stock market returns (m_i, p_{0i}, p_{20i}) and to the SSQ's with the two hypothetical gambles. To get these expected value of the latent individual-specific parameters conditional on

[20] Allowing for covariates (Appendix Table A6), ψ is estimated to be substantially less negative among more educated and wealthier people, indicating that their threshold value is closer to the nominal threshold zero.

the survey response and the statistical model, there are two steps. First, the distribution of the latent variables conditional on the observed responses can be obtained from the likelihood function using Bayes' theorem. Second, integrating out this function yields the individual-specific proxy variables ($\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\gamma}_i$) as the conditional expectations of the latent variables given the observed survey responses. These proxy variables deal with measurement error in survey responses. Appendix B spells out these steps in detail.

2. *Using individual-specific preference and belief parameters in regressions*

Our aim is to use the survey-based estimates of individual-specific parameters to explain heterogeneity in portfolio choice. Note that this proxy error is not conventional errors in variables. Because each proxy is a conditional expectation, which is basically a projection, this measurement error is uncorrelated with the proxy (and correlated with the true latent variable). Hence, it can be included on the right-hand side of a regression without creating an attenuation bias (see Kimball, Sahm and Shapiro, 2008).

When the regressions include other covariates as well the OLS estimates are unbiased if the proxies are estimated conditional on those covariates, too. We therefore estimate two sets of proxies. The first set is conditional on the survey answers to the risk tolerance and the stock market belief questions only. The second set is conditional on other covariates as well. We use the second set of proxy estimates as right-hand-side variables in regressions that also include those covariates. In the next section, we present such regressions to explain portfolio behavior based on our estimates of preferences and beliefs.

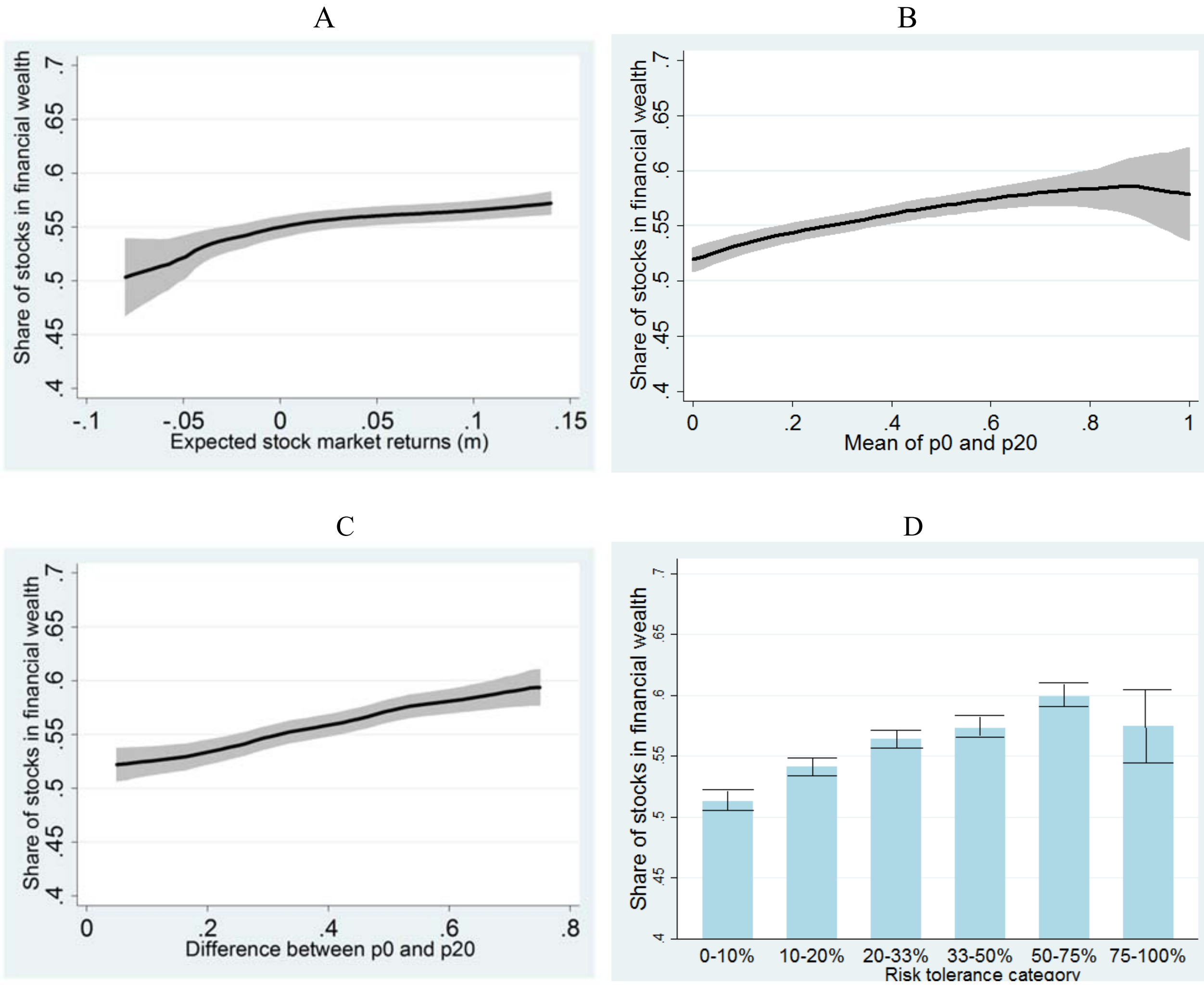
III. Explaining Heterogeneity in Portfolio Choice

A. Stock share and answers to survey questions

Before turning to the regressions based on our structural estimates of the latent preferences and beliefs, we investigate the relationship between the stock share of household portfolios and the raw survey responses. Figure 5 shows non-parametric regressions of the stock share in total financial assets on the survey answers to expected stock market returns (m_i), the average between the probability that the stock market would go up and that of an increase of 20 percent or more $((p0_i + p20_i)/2)$, the difference between those two ($p0_i - p20_i$), and the answer to the risk tolerance question with income level $100,000. (Figure A2 shows the analogous nonparametric regression results on $p0_i$ and $p20_i$ separately.)

The results indicate a positive relationship between the stock share of household portfolios and expected stock market returns (m_i) and the mean of the two probability responses $((p0_i + p20_i)/2)$. The stock share is also positively related to the difference between the responses to the probability questions ($p0_i - p20_i$), suggesting a negative relationship with perceived risk of stock returns. Finally, the stock share is monotonically positively related to the answers to the risk tolerance question except for the last categories that has relatively few responses, suggesting a monotonic positive relationship with risk tolerance. Hence, the relationship between the raw survey responses and the stock share has the direction benchmark theories of portfolio choice would suggest.

Figure 5. Stock Share versus Raw Survey Responses



We also estimate linear regressions with survey measure and the administrative measure of stock share as alternative left hand side variables and the same right hand-side variables entered with and without the control variables that include detailed measures of demographics, education, employment, income, wealth, as well as background risks of long-term care and longevity. The results are included in Tables A7 and A8 in the Appendix. The results imply statistically significant relationships of stock share with the survey answers with or without the control variables. The magnitudes of the associations are difficult to interpret because not all measures have a cardinal interpretation and because of the presence of survey noise. These problems are addressed in the next section.

B. Stock share and cardinal proxies of expectations and risk tolerance

Our more structural analysis has two goals. First, it relates the stock share of household portfolios to cross-sectional heterogeneity in preferences and expectations in a way that is related to portfolio choice theory thus making magnitudes easier to interpret. Second, it aims at incorporating survey noise in the estimation thus reducing its effect on the estimated magnitudes. This is a structural analysis in the sense that it makes use of additional assumptions in order to relate stock shares to heterogeneity in latent preferences and expectations. The analysis is still reduced form in the sense that it aims at uncovering associations without claims for causality. Nonetheless, since the explanatory variables are proxies that have cardinal interpretations relevant for economic theories, they potentially convey much more information than the relationship of raw survey responses to economic outcomes.

Start from a general function of the solution of optimal stock share

$$s_i^* = s^*\left(\mu_i, \sigma_i, \gamma_i, \kappa; x_i, u_i\right) \tag{9}$$

where μ_i and σ_i are the beliefs of person i about the mean and the standard deviation of one-year-ahead stock returns, γ_i is the parameter of risk tolerance, x_i is a vector of wealth, demographic variables and other risk factors that are measured in our data, and u_i combines all unobservables. We assume that unobservables are independent of observables.

The relative deviation of s^* *around* its mean value is related to relative deviations of the other variables around their mean values, holding values of x_i constant by

$$\frac{s_i^* - \overline{s^*}}{\overline{s^*}} \approx \beta_0 + \beta_1 \frac{\mu_i - \bar{\mu}}{\bar{\mu}} + \beta_2 \frac{\sigma_i - \bar{\sigma}}{\bar{\sigma}} + \beta_3 \frac{\gamma_i - \bar{\gamma}}{\bar{\gamma}} + \beta_4^{'} x_i + u_i. \qquad (10)$$

The coefficients approximate the first derivatives of the function around the mean values, with $\beta_1 = \partial \tilde{s}^* / \partial \tilde{\mu}$, $\beta_2 = \partial \tilde{s}^* / \partial \tilde{\sigma}$ and $\beta_3 = \partial \tilde{s}^* / \partial \tilde{\gamma}$, where the tilde denote relative differences from mean values. This approximation is a way of log-linearizing the function that allows observations with nonpositive values of some of the variables, which is relevant for μ_i in our case. We linearize about the risk tolerance parameter rather than relative risk tolerance to avoid the ambiguity that relative risk tolerance depends on the level of consumption.

We estimated (10) using the observed stock share s_i to approximate the target stock share s_i^*, and the individual proxies $\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\gamma}_i$ approximating the latent variables μ_i , σ_i and γ_i as described earlier. We estimated the equation by OLS both with and without covariates.[21] When we controlled for covariates in the stock share equation we entered the structural parameters that were estimated conditional on the same covariates. Kimball, Sahm, and Shapiro (2008) show that it is necessary to construct the proxies conditional on the same covariates as included in the main regression to deliver unbiased coefficient estimates. As the proxies are generated regressors, we estimated the standard errors by bootstrapping the entire estimation procedure including the structural estimation of the model underlying the proxies. We estimated two versions of each regression: one with the survey measure of the share of stocks in total financial wealth on the left hand side and one with the administrative measure of stock share in wealth held at Vanguard. The main results are in Table 5. Table A9 in the Appendix shows the detailed results.

[21] We do not use a Tobit-type procedure to account for the truncation at 0 and 1 because there are very few observations (less than 2 percent of the sample) at these boundaries.

The estimates show that the share of stocks is positively related to the perceived mean of stock market returns, negatively related to the perceived standard deviation of stock market returns, and positively related to the risk tolerance parameter. The estimated coefficients are statistically significant with the survey measure of stock shares on the left hand-side, but they are smaller and less significant in the administrative stock share regressions. In both cases the coefficients are very similar whether we enter them with or without the covariates.[22]

According to the point estimates, a one percent higher perceived mean is associated with one twentieth of a percent higher stock share; a one percent higher perceived standard deviation is associated with around one tenth of a percent lower stock share; and a one percent higher risk tolerance parameter is associated with one thirtieth of a percent higher stock share. Converting the relative magnitudes to absolute ones, our estimates imply that for the stock share to be higher by 1 percentage point expected returns need to be higher by 2.1 percentage points, the perceived standard deviation needs to be lower by 2.4 percentage points, or the risk tolerance parameter needs to be higher by 0.24.[23]

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[22] Table A9 in the Appendix shows that most of the coefficients on the other covariates are in line with prior expectations: the stock share is smaller in the employer-sponsored subsample and larger for wealthier and more educated individuals, especially for those with an MBA. Some of the other parameters are insignificant: the coefficient on the probability of needing long-term care (a factor of background risk), for example, is negative but insignificant.

[23] Comparing our estimates to the literature is not straightforward as most papers do not have cardinal proxies for the expectations and risk tolerance variables, and those that do estimate functional forms that are different from ours. Wherever we can make the comparison we find magnitudes that are very similar to our estimates. The closest to our specification are the estimates of Amromin and Sharpe (2012). On a sample of stockholders with positive expected returns they regress the log of the stock share on the log of their proxies of μ and σ. Their point estimates are +0.04 and -0.11, respectively. These magnitudes are very close to ours. The results of the Tobit model of Vissing-Jorgensen (2003), estimated on a sample of investors, imply that one percentage point higher returns expectations are associated with about 0.5 percentage point higher equity share. Kezdi and Willis (2011) estimate a coefficient of 0.3 in a truncated regression model estimated on a representative sample with stock shares on the left hand-side.

Table 5. Stock Shares versus Cardinal Proxies for Preferences and Beliefs

	Survey Stock Share		Administrative Stock Share	
	(1)	(2)	(3)	(4)
$\hat{\mu}_i$	0.058***	0.055***	0.052***	0.047***
	(0.010)	(0.009)	(0.008)	(0.008)
$\hat{\sigma}_i$	-0.093*	-0.083	-0.068	-0.083*
	(0.046)	(0.051)	(0.040)	(0.038)
$\hat{\gamma}_i$	0.034***	0.033**	0.012	0.013
	(0.009)	(0.010)	(0.010)	(0.010)
constant	-0.001	1.136	-0.001	0.803
	(0.008)	(0.649)	(0.007)	(0.519)
covariates	N	Y	N	Y
R^2	0.019	0.045	0.013	0.038
N	4414	4414	4414	4414

Notes. Stock share in total financial wealth (survey measure) and in Vanguard accounts (administrative measure) are regressed on proxies for the expected stock returns ($\hat{\mu}_i$), perceived standard deviation of stock returns ($\hat{\sigma}_i$), and the parameter of risk tolerance ($\hat{\gamma}_i$). All variables are expressed as relative differences normalized to their mean values (as specified in equation (10)). Control variables: married, male, age, whether respondent comes from the employer-sponsored subsample, education (below college; college; MBA; PhD, other higher degree); log financial wealth, log wage, dummy for owning a house, log annuity income (Social Security and DB pensions) for retired, log expected annuity income for non-retired; dummy for retired, log home stock; subjective probability of needing long-term care, and longevity expectations.
Bootstrap standard errors in parentheses. *, **, and *** implies significance at 5%, 1% and 0.1% level, respectively.

Table 6 shows results of analogous estimations that do not take care of measurement error in the survey answers (Table A10 in the Appendix contains all results). Instead of the cardinal proxies $\hat{\mu}_i, \hat{\sigma}_i, \hat{\gamma}_i$, these regressions include the raw survey answers to the stock market expectation question (m_i), a crude transformation of the probability answers to approximate

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Our log-linearized estimates imply that, around its mean, a one point difference in μ is associated with a 0.45 percentage point difference in stock shares. In a Tobit model of stock shares that combines the extensive and intensive margins Kimball, Sahm and Shapiro (2008) find a small magnitude for the association with the cardinal proxy of risk tolerance.

perceived risk,[24] and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions (κ set to zero). The coefficient estimates are qualitatively similar to the baseline results reported in Table 5 above, but the magnitudes are considerably attenuated. The absolute values of the point estimates are one third to one half of the baseline estimates. These results are consistent with substantial measurement error in the raw survey answers. They show the importance of taking into account measurement error in the construction of the proxies and in using them in econometric models.

Table 6. Stock Shares Versus Error-Ridden Cardinal Measures of Preferences and Beliefs. Estimation without taking care of measurement error in the cardinal proxies.

	Survey Stock Share		Administrative Stock Share	
	(1)	(2)	(3)	(4)
m_i	0.017***	0.020***	0.020***	0.021***
	(0.004)	(0.004)	(0.004)	(0.004)
$\tilde{\sigma}_i$	-0.029***	-0.019**	-0.025***	-0.020**
	(0.006)	(0.007)	(0.006)	(0.006)
$\tilde{\gamma}_i$	0.021***	0.020***	0.013**	0.013**
	(0.005)	(0.005)	(0.004)	(0.004)
constant	-0.001	1.120*	-0.000	0.781
	(0.007)	(0.565)	(0.006)	(0.507)
covariates	N	Y	N	Y
R^2	0.013	0.039	0.012	0.038
N	4414	4414	4414	4414

Notes. Left-hand-side variables and covariates as in Table 5. Main right-hand-side variables are the raw survey answers to the stock market expectation question (m_i), a crude transformation of the probability answers to approximate perceived risk ($\tilde{\sigma}_i = 0.2 / \left(\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i})\right)$), and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions (κ set to zero).
Bootstrap standard errors in parentheses. *, **, and *** implies significance at 5%, 1% and 0.1% level, respectively.

[24] $\tilde{\sigma}_i = 0.2 / \left(\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i})\right)$. The denominator replaced with 0.2 if zero to obtain $\tilde{\sigma}_i = 1$, which is larger than the maximum of all other values. This imputation affects less than 10% of the observations. Alternative imputations that replace the denominator with other values yield very similar estimates.

Table A11 shows estimates analogous to our benchmark model presented in Table 5 above for the employer-sponsored subsample. Self-selection to Vanguard is arguably substantially less severe in this subsample. However, the differences are small, suggesting that selection bias is unlikely to have a substantial effect on our main results.[25]

C. Interpreting the magnitudes

How might one evaluate the estimates relative to an economic model? The simplest model of Merton (1969) with CRRA utility would imply that the coefficient on log μ should be 1, the coefficient on log σ should be -2, and the coefficient on log γ should be 1 again. The same implications hold if we modify the utility function in the Merton model to incorporate the subsistence level of consumption as in equation (1) above.

The *relative* magnitudes of the estimated coefficients report in Table 5 are remarkably close to these theoretical implications of the Merton benchmark. In the regressions on the survey measure of stock share, the coefficient on the (approximately log-linearized) expected value and risk tolerance proxies are close to each other, and the coefficient on the standard deviation proxy is close to be negative two times their magnitudes. At the same time, the magnitudes are indeed smaller than in the benchmark model: each estimate is about one twentieth of what a simple theory implies.

In principle, the attenuation bias may arise from classical errors in variables on the right hand-side or appropriate non-classical errors in the left hand-side variable. Recall that our

[25] Tables A12 through A15 in the Appendix show that results from analogous regressions are very similar in various subsamples, such as the sample of individuals that joined Vanguard with their private accounts, the sample of individuals with high share of household wealth held at Vanguard, and the sample of individuals with directly held stocks.

measures of beliefs and preferences already take care of substantial survey noise that arise from noisy responses conditional on the latent variables. While it is of course possible for those latent variables to exhibit additional noise, due to, for example, mood effects, that noise would have to be extremely large for the observed attenuation. We believe that the magnitude of the attenuation and its similar strength across the coefficients call for an explanation beyond these measurement issues.

We can represent the substantially attenuated response of stock holding to beliefs and preferences by expressing observed stock shares as a linear combination of the individual optimum s_i^* and the average stock share $\overline{s}$ plus additional heterogeneity

$$s_i = \lambda s_i^*\left(\mu_i, \sigma_i, \gamma_i\right) + \left(1 - \lambda\right)\overline{s} + v_i \tag{11}$$

where λ is the weight on the individual optimum given beliefs and preferences, (1- λ) is the weight on the average stock share, and v_i is heterogeneity in stock shares due to other factors. This model can be viewed as a simple statistical representation of the attenuation. It can be also interpreted as a behavioral model, in which investors consider the possibility that everyone else may choose the right stock share even if their own beliefs and preferences imply a different choice, and their decision combines the two.[26] Such behavior could account for the finding we discussed earlier that those who report negative expected returns in the survey continue to hold stock and those who are very optimistic do not have extreme exposure to the stock market.

[26] A possible reason for this behavior is mean reversion of beliefs combined with a strong inertia in portfolio choice. Some individuals who expect extreme returns currently think returns will revert back to more normal in the future, but do not make the high-frequency adjustments to their portfolios to align with current extreme expectations.

Expressing equation (11) in deviations from averages, denoting the coefficients of the log-linearized optimal stock share by β^0 and decomposing heterogeneity due to other factors into observed and unobserved parts yields

$$\frac{s_i-\bar{s}}{\bar{s}} \approx \beta_0+\lambda\beta_1^0\frac{\mu_i-\bar{\mu}}{\bar{\mu}}+\lambda\beta_2^0\frac{\sigma_i-\bar{\sigma}}{\bar{\sigma}}+\lambda\beta_3^0\frac{\gamma_i-\bar{\gamma}}{\bar{\gamma}}+\beta_4^{'}x_i+u_i. \tag{12}$$

This is a constrained version of equation (10), with the Merton solution implying $\beta_1^0=1$, $\beta_2^0=-2$, $\beta_3^0=1$. We estimate the constrained versions of each unconstrained regression presented in Table 5 above. Table 7 shows the results. The estimated λ is around 0.05 when the left hand side variable is the survey measure of stock share, and it is around 0.03 when the left hand side variable is the administrative measure of stock share. The proportionality restriction holds reasonably well in the data as one would expect from inspection of the results in Table 5. The Wald test does not reject the null of proportionality for the survey data, but does marginally for administrative data (more so without covariates). These results suggest strong attenuation in the association of stock shares with beliefs and preferences, but that the degree of attenuation is well represented by a single factor, as expressed by equation (11).

The forgoing model of attenuation assumes that individuals down-weight both their preferences and beliefs in favor of the market average. A perhaps less radical behavioral model is that individuals keep to their preferences, but moderate their reactions to returns beliefs. In Appendix Table A16 we present such estimates by excluding the preference measure from the s_i^*. The estimated λ increases slightly to 5 or 6 percent and the proportionality restrictions are far from the rejection region.

Table 7. Observed stock shares and theoretically optimal stock shares

	Survey Stock Share		Administrative Stock Share	
	(1)	(2)	(3)	(4)
λ	0.046***	0.045***	0.032***	0.032***
	(0.006)	(0.006)	(0.006)	(0.007)
covariates	N	Y	N	Y
R^2	0.017	0.044	0.010	0.036
N	4414	4414	4414	4414
p-value of Wald test on restriction	0.240	0.258	0.010	0.033

Notes. Regression results from equation (12) imposing $\beta_1^0 = 1$, $\beta_2^0 = -2$, and $\beta_3^0 = 1$.

Bootstrap standard errors in parentheses. *, **, and *** implies significance at 5%, 1% and 0.1% level, respectively.

D. Alternative Benchmark Model

The Merton model is a simple and useful benchmark. However, it has assumptions that are very far from the way people invest in our sample as it requires continuous rebalancing, no background risk, and it allows for unlimited leverage and short sales. We therefore investigate whether adding these realistic features would move the predictions of the benchmark model more in line with what we observe in the data.

The model we use is a life cycle portfolio choice model in discrete time, with consumption and investment decisions at the yearly frequency and portfolio shares constrained to be between zero and one. The model incorporates background risk in income and longevity. Similarly to the Merton model, it has a risky asset (stocks) and a risk-free asset (bonds), and it is a model of demand, taking returns on those assets as given. It has no closed-form solution and is solved recursively. Our model can be viewed as a generalization of the model of Cocco, Gomes and Maenhout (2005). Appendix C contains its details and main results.

The lifecycle model of portfolio choice implies magnitudes that are similar to the Merton model in the range of interior solutions, and it implies corner solutions of zero and 100 percent stock shares otherwise. Our estimates of these associations are substantially weaker. Stock share is close to fifty percent among those who expect non-positive stock returns in our sample, and it is substantially less than 100 percent on average among people with very high expectations even with low perceived risk and low risk tolerance. In the theoretical model, the share of stocks increases from zero to 100 percent if expected returns raise from the risk-free rate to seven percentage points above the risk-free rate even for investors that are in the top third of our estimated distribution of risk aversion and perceived risk. Our estimates imply that a seven percentage point difference in expected returns corresponds to less than a four percentage point difference in stock shares. Again, our estimate is about one twentieth of the magnitude implied by theory. Differences between the model and our estimates with respect to the role of perceived risk and risk tolerance are similarly large.

We investigate the attenuation of our estimates compared to the life cycle portfolio choice model by estimating equation (11) with the solution implied by this model for s_i^*, estimated for each individual using a linearized version of the model solution. The result is $\lambda = 0.043$ when the left hand side variable is the survey measure of stock share and we do not include other covariates. This result is very similar to our estimate based on the Merton solution, as presented in column (1) in Table 7.

Risk preferences, expected stock returns, and the risk of stock returns are fundamental elements of any portfolio choice model. People should not hold stocks if they think their returns will be lower than returns on risk-free assets, and they should avoid or embrace risk in their investment decisions in a way that is in line with their choices in other gambles with money.

These features are implied both by the very stylized Merton model and our more realistic life cycle model, yet our estimates do not deliver them. The two portfolio choice models imply similar magnitudes in the range of an interior solution without short sales and leverage, and our estimates fall short of those magnitudes.

IV. Conclusion

There is substantial heterogeneity in portfolio decisions across households. This paper uses a distinctive measurement and analytic strategy that combines high-quality measurement of portfolio shares, preferences about risk, and beliefs about returns in an attempt to explain this heterogeneity. The approach uses purposely-constructed measures to elicit measures of preferences and beliefs that have quantitative interpretations. This paper does find that risk preference and moments of the subjective returns distribution—both mean and variance—do have a role in understanding why portfolio choices differ. That the survey measures of preference and belief do align with portfolio choices provides external validation of our approach to measuring them.

The size of the estimated associations of the risk and belief parameters is, nonetheless, substantially smaller in magnitude than benchmark theories would suggest. We call this finding the "attenuation puzzle." Our methods produce risk and belief parameters that measure the precise, quantitative variables that should explain portfolio choice. Hence, the attenuation cannot be dismissed because the measures of preference and belief are only loosely related to what should drive portfolio choice. Moreover, the statistical procedure deals with measurement error in these parameters, which is the other most obvious source of such attenuation. The econometric procedure estimates the response error in the survey measures of preferences and beliefs and produces individual-specific preference and belief parameters that are immune from the standard effects of using noisy estimates as regressors.

We argue that the selected nature of the sample is unlikely to account for the small magnitudes. The results are nearly identical for the subset of the sample who came to Vanguard via their employer's choices, so individual selection to use Vanguard as a provider is not driving the findings.

Another explanation for the attenuated response could be background risk that would reduce stockholding given the perception of riskiness of stock returns per se. This source of attenuation, however, mostly affects the coefficient of our measure of perceived risk, and it should not lead to substantial bias in the estimates of the coefficients of risk tolerance or perceived expected returns.

The attenuated response of stock shares to beliefs and preferences is responsible for observing individuals who think that stocks are dominated in return often holding a high fraction of their portfolio in stock. Symmetrically, this attenuated response makes many individuals who think that stocks have very high returns hold too little stocks. What might account for such behavior? Our finding in Figure 2 that stock shares are very static suggests an answer: It is possible that people make their portfolio choice decision very infrequently, much less frequently than the annual rebalancing of the theory in Section III. If household are cognizant of this inertia and if they feel that their preferences and beliefs might change, then it would make sense to have a damped response to them. We show that the estimated behavior is indeed consistent with individuals mixing their own preferences and beliefs with the market average behavior. While far from fully worked out as an explanation, our finding that there is a coherent but attenuated response to the vector of risk tolerance, mean return, and variance of return does perhaps point toward such explanations. Similarly, the deviations of the survey and administrative measures of portfolio shares suggest that many respondents do not follow their portfolios closely. If they are aware of this, that might well be a good reason for them to choose a portfolio closer to what the representative individual would choose than the one they would choose based on their preferences or beliefs at a particular moment.

The results are consistent with decision makers mixing their own risk preference and beliefs about stock market returns with the preferences and beliefs of the representative consumer. On the one hand, people may follow advice or buy what is offered despite their preferences and beliefs. On the other hand, changes in beliefs may not translate to changes in portfolio composition because of inertia, due to inattention or fixed costs. While our data do not allow us to sort out these explanations, our paper makes substantial progress by quantifying the role of individual risk preferences and beliefs about stock market returns in the heterogeneity in portfolio choice across households, using high-quality and precise measurements of preferences and beliefs.

References

Ameriks, John, Joseph Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti (2015a): "Long-Term Care Utility and Late in Life Saving," Vanguard Research Initiative Working Paper.

Ameriks, John, Joseph Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti (2015b): "Long-Term Care Insurances, Annuities, and the Under-Insurance Puzzle," Work in progress.

Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn Van Nieuwerburgh (2011): "The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives," *Journal of Finance*, 66(2), pages 519–561.

Ameriks, John, Andrew Caplin, Minjoon Lee, Matthew D. Shapiro, and Christopher Tonetti (2014a): "The Wealth of Wealthholders," Vanguard Research Initiative Working Paper.

Ameriks, John, Andrew Caplin, Minjoon Lee, Matthew D. Shapiro, and Christopher Tonetti (2014b): "Vanguard Research Initiative: Survey 1 Documentation and Tabulations," Vanguard Research Initiative Working Paper.

Amromin, Gene and Steven A. Sharpe (2012): "From the Horse's Mouth: How do Investor Expectations of Risk and Return Vary with Economic Conditions?" *Federal Reserve Bank of Chicago Working Paper*.

Armantier, Olivier, Wändi Bruine de Bruin, Simon Potter, Giorgio Topa, Wilbert van der Klaauw, and Basit Zafar (2013): "Measuring Inflation Expectations," *Annual Review of Economics*, 5, 273-301.

Barsky, Robert B., F. Thomas Juster, Miles S. Kimball, and Matthew D. Shapiro (1997): "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics*, 112, 537-579.

Bruine de Bruin, Wändi, Charles F. Manski, Giorgio Topa, and Wilbert van der Klaauw (2011): "Measuring Consumer Uncertainty about Future Inflation," *Journal of Applied Econometrics*, 26, 454–478.

Brunnermeier, Markus, and Stefan Nagel (2008): "Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-Evidence on Individuals' Asset Allocation," *American Economic Review*, 98, 713-736.

Campbell, John Y. (2006): "Household Finance," *Journal of Finance* 61, 1553-1604.

Calvet, Laurent E. and Paolo Sodini (2014): "Twin Picks: Disentangling the Determinants of Risk-Taking in Household Portfolios," *Journal of Finance* 69(2), 867-906.

Cocco, Joao F., Francisco J. Gomes, and Pascal J. Maenhout (2005): "Consumption and Portfolio Choice over the Life Cycle," *Review of Financial Studies*, 18, 491-533.

Dimmock, Stephen G., Roy Kouwenberg, Olivia S. Mitchell and Kim Peijnenburg (2013): "Ambiguity Aversion and Household Portfolio Choice: Empirical Evidence," *NBER Working Paper* 18734.

Dohmen, Thomas, Armin Falk, David Huffman, and Uwe Sunde (2010): "Are Risk Aversion and Impatience Related to Cognitive Ability?" *American Economic Review,* 100, 1238–1260.

Dohmen, Thomas, Armin Falk, David Huffman, Uwe Sunde, Jürgen Schupp, and Gert G. Wagner (2011): "Individual risk attitudes: Measurement, determinants, and behavioral consequences," *Journal of the European Economic Association* 9 (3), 522-550.

Dominitz, Jeff and Charles F. Manski (2007): "Expected Equity Returns and Portfolio Choice: Evidence from the Health and Retirement Study." *Journal of the European Economic Association* 5(2-3), 369-379.

Glaser, Markus and Martin Weber (2005): "September 11 and stock return expectations of individual investors." *Review of Finance* 9 (2), 243–279.

Glaser, Markus, T. Langer, J. Reynders and Martin Weber (2007): "Framing effects in stock market forecasts: The difference between asking for prices and asking for returns." *Review of Finance*, 11, 325–357.

Greenwood, Robin and Stefan Nagel (2009): "Inexperienced Investors and Bubbles," *Journal of Financial Economics,* 93 (2), 239-258.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales (2014): "Time Varying Risk Aversion," *Working paper*.

Hoffman, Arvid O. I., Thomas Post and Joost M. E. Pennings (2013): "Individual investor perceptions and behavior during the financial crisis," *Journal of Banking & Finance*, 37, 60–74.

Hudomiet, Peter, Gabor Kezdi and Robert J. Willis (2011): "Stock Market Crash and Expectations of American Households," *Journal of Applied Econometrics,* 26, 393-415.

Hurd, Michael D. (2009): "Subjective Probabilities in Household Surveys," *Annual Review of Economics*, 1, 543-562.

Hurd, Michael D., Maarten van Rooij, and Joachim Winter (2011): "Stock Market Expectations of Dutch Households," *Journal of Applied Econometrics,* 26, 416-436.

Josef, Anika K., David Richter, Gregory R. Samanez-Larkin, Gert G. Wagner, Ralph Hertwig, Rui Mata (2016): "Stability and Change in Risk-Taking Propensity Across the Adult Life Span," *Journal of Personality and Social Psychology,* Advance online publication. http://dx.doi.org/10.1037/pspp0000090.

Kezdi, Gabor and Robert J. Willis (2011): "Household Stock Market Beliefs and Learning," *NBER Working Paper* 17614.

Kimball, Miles S., Claudia R. Sahm, and Matthew D. Shapiro (2008): "Imputing Risk Tolerance from Survey Responses," *Journal of American Statistical Association*, 103, 1028-1038.

Lönnqvist, Jan-Erik, Markku Verkasalo, Gari Walkowitz, and Philipp C. Wichardt (2015): "Measuring individual risk attitudes in the lab: Task or ask? An empirical comparison," *Journal of Economic Behavior and Organization.* 119, 254–266.

Malmendier, Ulrique and Stefan Nagel (2011): "Depression Babies: Do Macroeconomic Experiences Affect Risk-Taking?" *Quarterly Journal of Economics*, 126, 373-416.

Malmendier, Ulrike, and Stefan Nagel (2015): "Learning from Inflation Experiences," *Quarterly Journal of Economics*, forthcoming.

Manski, Charles F. (2004): "Measuring Expectations." *Econometrica*, 72, 1329-1376.

Merkle, Christoph, and Martin Weber (2014): "Do investors put their money where their mouth is? Stock market expectations and investing behavior," *Journal of Banking & Finance*, 46, 372–386.

Merton, Robert C. (1969): "Lifetime Portfolio Selection under Uncertainty: the Continuous-Time Case," *Review of Economics and Statistics*, 51, 247-257.

Merton, Robert C. (1980): "On Estimating the Expected Return on the Market: an Exploratory Investigation." *Journal of Financial Economics*, 8, 323-361.

Piazzesi, Monika and Martin Schneider (2009): "Momentum Traders in the Housing Market: Survey Evidence and a Search Model," *American Economic Review*, 99, 406-11

Seru, Amit, Tyler Shumway and Noah Stoffman (2010): "Learning By Trading," *Review of Financial Studies*, 23 (2): 705-739..

Vissing-Jorgensen, Annette. (2003): "Perspectives on Behavioral Finance: Does "Irrationality" Disappear with Wealth? Evidence from Expectations and Actions," *NBER Macroeconomics Annual*, 2003, 139-194.

Wachter, Jessica A. and Motohiro Yogo (2010): "Why Do Household Portfolio Shares Rise in Wealth?" *Review of Financial Studies*, 23, 3929–3965.

Weber, Martin, Elke U. Weber and Alen Nosic (2013): "Who takes risks when and why: Determinants of changes in investor risk taking," *Review of Finance*, 17(3), 847-883.