Vanguard Research Initiative Technical Report: Long-Term-Care Strategic Survey Questions

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We develop four strategic survey questions (SSQs) to estimate preference parameters related to the coefficient of relative risk aversion, expenditure when in need of help with activities of daily living, and bequests. In this appendix we present the text of the SSQs, the distribution of responses to these questions, and the optimal responses to strategic survey questions (as functions of preference parameters) given utility functional forms. Respondents provided answers to nine variants of four different types of SSQs: two variants of SSQ 1, three variants of SSQ 2, three variants of SSQ 3, and a single variant of SSQ 4. A summary of these questions and their variants is provided in Table 1. For each SSQ, there is a sequence of comprehension questions that verifies respondent understanding of the scenarios just prior to requesting the answer. There are between two and nine questions in each sequence. If answered incorrectly, the respondents are twice asked to correct incorrect answers, and if then still answered incorrectly, given the correct answer. For further documentation of the VRI surveys, including a dynamic link to the survey instrument, see `http://ebp-projects.isr.umich.edu/VRI/survey_overview.html`.

	Question	Objective	Scenario Parameters	Preference Parameters
SSQ 1	Lottery over spending	$\lambda^* : \frac{1}{1-\gamma}(W)^{1-\gamma} = \frac{0.5}{1-\gamma}(2W)^{1-\gamma} + \frac{0.5}{1-\gamma}((1-\lambda^*)W)^{1-\gamma}$	(a) $W = \$100$K (b) $W = \$50$K	γ
SSQ 2	Allocation between ordinary and ADL states	$\max_{z_1,z_2} \quad \pi\frac{z_1^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(\theta_{ADL})^{-\gamma}(z_2+\kappa_{ADL})^{1-\gamma}}{1-\gamma}$	(a) $W = \$100$K, $\pi = 0.75$ (b) $W = \$100$K, $\pi = 0.50$ (c) $W = \$50$K, $\pi = 0.75$	γ, θ_{ADL}, κ_{ADL}
SSQ 3	Allocation between ADL and bequest states	$\max_{z_1,z_2} \quad (\theta_{ADL})^{-\gamma} \ \frac{(z_1+\kappa_{ADL})^{1-\gamma}}{1-\gamma} + (\theta_{beq})^{-\gamma} \ \frac{\left(z_2+\kappa_{beq}\right)^{1-\gamma}}{1-\gamma}$	(a) $W = \$100$K (b) $W = \$150$K (c) $W = \$200$K	γ, θ_{ADL}, κ_{ADL} θ_{beq}, κ_{beq}
SSQ 4	Indifference between public and private LTC	$W^* : (\theta_{ADL})^{-\gamma} \ \frac{(\psi_G+\kappa_{ADL})^{1-\gamma}}{1-\gamma} + (\theta_{beq})^{-\gamma} \ \frac{\left(W^*+\kappa_{beq}\right)^{1-\gamma}}{1-\gamma} = (\theta_{ADL})^{-\gamma} \ \frac{(z_1+\kappa_{ADL})^{1-\gamma}}{1-\gamma} + (\theta_{beq})^{-\gamma} \ \frac{\left(W^*-z_1+\kappa_{beq}\right)^{1-\gamma}}{1-\gamma}$	(a) Public Care Available	γ, θ_{ADL}, κ_{ADL} θ_{beq}, κ_{beq}, ψ_G

Table 1: **Link between parameters and SSQs:** The first column briefly summarizes the tradeoffs, while the second lists the underlying optimization problem. The third column lists how question parameters were changed for different variations of each SSQ, where W is wealth and $1-\pi$ is the probability of needing LTC. The z_1 in SSQ 4 is the optimal z_1 function calculated in SSQ 3. The fourth column lists the parameters that determine optimal responses in the model.

1 SSQ 1

The next section asks you to think about your **willingness to take financial risk**.

We will ask you to imagine a situation. We will ask you questions about the situation. We will then ask you about choices that you would make. Even if it is hard to imagine yourself in this situation, please try your best. Finally we will ask you how changes in the situation affect your choices.

We are interested in your preferences between having a set amount of guaranteed money and taking a risk that might increase or decrease the amount of money you have available to spend.

Suppose you are 80 years old. Suppose, further, that for the next year:

- You live alone, rent your home, and pay all your own bills.
- You are in good health and will remain in good health.
- You will have no medical bills or other unexpected expenses.
- You do not work.

You must decide between two plans for the amount you will have available to spend next year.

- Plan A guarantees that you will have $100,000 for spending next year.
- Plan B will possibly provide you with more money, but is less certain. There is a 50% chance Plan B would double your money, leaving you with $200,000, and a 50% chance that it would cut it by a third, leaving you with $67,000.

The plan you choose will determine how much you have to spend for the next year. This choice affects your finances only for next year, not for any years after that. At the end of next year you will again be offered the same choice with another $100,000.

The rules are as follows:

- You have no other assets or income, and so this is the only money you have available for all your spending next year from either Plan A or Plan B.
- Any money that is not spent at the end of next year cannot be saved for the future.
- You cannot give any money away or leave it to others in your will.
- If you need anything next year, you have to pay for it. No one else can buy anything for you.

The comprehension questions discussed in the introduction are presented at this point (available on pages 29–32 of Survey 2). The respondents are then given the following screen and asked to record their decision.

Now we will ask you to choose between the two plans.

As a reminder, suppose you are 80 years old. Suppose, further, that for the next year:

- You live alone, rent your home, and pay all your own bills.
- You are in good health and will remain in good health.
- You will have no medical bills or other unexpected expenses.
- You do not work.

You must decide between two plans for the amount you will have available to spend next year.

- Plan A guarantees that you will have the $100,000 for spending next year.
- Plan B will possibly provide you with more money, but is less certain. There is a 50% chance Plan B would double your money, leaving you with $200,000, and a 50% chance that it would cut it by a third, leaving you with $67,000.

Would you choose Plan A or Plan B?

The respondents are then given a series of questions with different amounts risked until they choose Plan A once and Plan B once, or until they reach the extreme cases of either choosing Plan A when risking only 10% of income or Plan B when risking 75% of income. These bounds and branching logic mirror the six risk tolerance categories of Barsky, Juster, Kimball, and Shapiro (1997) used in the HRS since 1994. See Kimball, Sahm, and Shapiro (2008), Table 1.

Depending on the bracket in which their answers fall, they are asked some variant the following question and indicate their willingness to risk income using the slider discussed in Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2017) and Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2018).

You have indicated that:

- You would choose Plan A when choosing between two plans, the first of which guarantees $100,000 is available as spending next year and the second of which offers a 50% chance that your money would double to $200,000 and a 50% chance that it would be cut by 33% to $67,000.
- You would choose Plan B when choosing between two plans, the first of which guarantees $100,000 is available as spending next year and the second of which offers a 50% chance that your money would double to $200,000 and a 50% chance that it would be cut by 20% to $80,000.

What is the largest percent of your money that you would be willing to risk and still choose Plan B?

2 SSQ 2

The next section asks you to think about **long term care**. Again, we will ask you to imagine a situation and describe the rules that apply. We will ask you questions about the situation. We will then ask you about choices that you would make. Even if it is hard to imagine yourself in this situation, please try your best. Finally we will ask you how changes in the situation affect your choices.

We are interested in how you trade off your desire for resources when you do and when you do not need help with activities of daily life (ADLs). This scenario is hypothetical and does not reflect a choice you are likely ever to face.

Suppose you are still 80 years old, live alone, rent your home, and pay all your own bills. Now, suppose that there is a chance that you will need help with ADLs in the next year. If you need help with ADLs you will need long-term care.

- There is a **25%** chance that you **will** need help with ADLs for all of next year.
- There is a **75%** chance that you **will not** need any help at all with ADLs for all of next year.

You have **$100,000** to divide between two plans for the next year. This choice will affect your finances for next year alone. At the end of next year you will be offered the same choice with another $100,000 for the following year.

- Plan C is hypothetical ADL insurance that gives you money if you **do** need help with ADLs.
  - For every $1 you put in Plan C, you will get **$4** to spend if you need help with ADLs.
  - From that money, you will need to pay all your expenses including long-term care at home or in a nursing home and any other wants, needs, and discretionary purchases.
- Plan D gives you money only if you **do not** need help with ADLs.
  - For every $1 you put in Plan D, you will get **$1** to spend if you **do not** need help with ADLs.
  - From that money, you will need to pay for all of your wants, needs, and discretionary purchases.

Here are the rules for this scenario.

- You can only spend money from Plan C or Plan D next year. You do not have any other money.
- If you want to be able to spend whether or not you need help with ADLs, you need to put money into both plans.
- If you need help with ADLs, all money in Plan D is lost.
- If you do not need help with ADLs, all money in Plan C is lost.
- Any money that is not spent at the end of next year cannot be saved for the future, be given away, or be left as a bequest

- You must make your choice before you know whether you need help with ADLs. Once you make your choice, you cannot change how you split your money.
- Regardless of whether or not you need help with ADLs, your hospital, doctor bills, and medications are completely paid by insurance.
- Other than Plan C, you have no other resources available to help with your long-term care. **You** have to pay for any long-term care you may need from Plan C.
- There is **no public-care option or Medicaid** if you do not have enough money to pay for a nursing home or other long-term care.
- An impartial third party that you trust will verify whether or not you need help with ADLs immediately, impartially, and with complete accuracy.

The comprehension questions discussed in the introduction are presented at this point (available on pages 44–48 of Survey 2). The respondents were then given the following screen and asked to record their decision.

Again, suppose you are 80 years old, live alone, rent your home, and pay all your own bills.

- There is a **25%** chance that you **will** need help with ADLs for all of next year.
- There is a **75%** chance that you **will not** need any help at all with ADLs for all of next year.

You have **$100,000** to divide between two plans for the next year.

- Plan C is hypothetical ADL insurance that gives you money if you **do** need help with ADLs. For every $1 you put in Plan C, you will get **$4** to spend if you need help with ADLs.
- Plan D gives you money only if you **do not** need help with ADLs. For every $1 you put in Plan D, you will get **$1** to spend if you **do not** need help with ADLs.

The next page will ask about your choices.

They are then given the following screen where they can record their optimal allocation using the slider.

Please make your decision on splitting money into Plan C and Plan D by clicking on the scale below. To put more money in Plan C, move the slider to the right. To put more money in Plan D, move the slider to the left. The numbers in the box will change as you move the slider to let you know how much you will receive if you need long-term care and if you do not.

Please move the slider to see how it works. When you are ready, place the slider at the split you want and click NEXT to enter your choice.

3 SSQ 3

The next section asks you to think about **bequests and long term care**.

Again, we will ask you to imagine a situation and describe the rules that apply. We will ask you questions about the situation. We will then ask you about choices that you would make. Even if it is hard to imagine yourself in this situation, please try your best. Finally we will ask you how changes in the situation affect your choices.

We are now going to ask about a different situation where you are older and definitely need long-term care. In this situation, you are asked to make tradeoffs between spending on your long-term care and leaving a bequest. This scenario is hypothetical and does not reflect a choice you are likely ever to face.

Suppose you are 85 years old, live alone, rent your home, and pay all your own bills. You know with certainty that you will live for only 12 more months and that you will need help with ADLs for the entire 12 months.

You have **$100,000** that you need to split into Plan E and Plan F.

- Plan E is reserved for your spending. From Plan E, you will need to pay all of your expenses, including long-term care and any other wants, needs, and discretionary purchases.
- Plan F is an irrevocable bequest.

Here are the rules for this scenario.

- You have no money other than the $100,000.
- Other than Plan E, you have no other resources available to help with your long-term care. **You** have to pay for any long-term care you may need from Plan E.
- No one—including friends or family—can take care of you for free. Long-term care must be purchased at market rates.
- Any money in Plan E that you do not spend cannot be given away or left as a bequest.
- Bequests from Plan F are not subject to any taxation.
- Once you make your choice of plans, you cannot change how you split your money.
- You have full insurance that covers all of your hospital, doctor, and medications, but you have no long-term care insurance.
- There is **no public-care option or Medicaid** if you do not have enough money to pay for a nursing home or other long-term care.

The comprehension questions discussed in the introduction are presented at this point (available on pages 52–54 of Survey 2). The respondents were then given the following screen and asked to record their decision.

> We now would like you to divide $100,000 between the two plans.
>
> Again suppose you are 85 years old, live alone, rent your home, and pay all your own bills. You know with certainty that you will live only 12 more months, and that you will need help with ADLs for the entire 12 months.
>
> You have **$100,000** that you need to split into Plan E and Plan F.
>
> - Plan E is reserved for your spending. From Plan E, you will need to pay all your expenses including long-term care at home or in a nursing home and any other wants, needs, and discretionary purposes.
> - Plan F is an irrevocable bequest.

They are then given the following screen where they can record their optimal allocation using the slider.

> Please make your decision on splitting money into Plan E and Plan F by clicking on the scale below. To put more money in Plan E, move the slider to the right. To put more money in Plan F, move the slider to the left. The numbers in the box will change as you move the slider to let you know how much you will have to spend and how much you will leave as a bequest.
>
> Please move the slider to see how it works. When you are ready, place the slider at the split you want and click NEXT to enter your choice.

4 SSQ 4

> The next section asks you to think about **publicly-funded long term care**.
>
> Suppose you are 85 years old, live alone, rent your home, and pay all your own bills. You know with certainty that you will live only 12 more months, and that you will need help with ADLs for the entire 12 months.
>
> The final scenario is identical to the previous scenario **except you are entitled to an option of a publicly-funded nursing home**. You can now legally leave **all** your assets as bequests and live in a publicly-funded nursing home for a year.
>
> In this scenario, you have **$100,000** and must decide between either Plan G or Plan H.

- Plan G puts **all** of your $100,000 in an irrevocable bequest, and you will live in a publicly-funded nursing home.
- Plan H allows you to split your $100,000 between spending and bequests, and you are not eligible to live in a publicly funded nursing home.
  - From Plan H, you must designate whether to leave an irrevocable bequest, and if so, how much.
  - From Plan H funds you do not designate as a bequest, you will need to pay all your expenses, including long-term care and any other wants, needs, and discretionary purchases.

Here are the rules for this scenario.

- You have no money other than the $100,000.
- Once you make your choice, you cannot change it.
- You have full insurance that covers all of your hospital, doctor, and medication costs, but you have no private long-term care insurance.
  - If you choose Plan G, you are entitled to care in a publicly-funded nursing home.
  - If you choose Plan H, you will need to pay for all long-term care expenses.
- If you choose Plan H, you must choose the size of your bequest from the $100,000. The size of the bequest will be determined at the start of the year, but not available to recipients until the end of the year.
- The publicly-funded nursing home has similar range of quality and choice as nursing homes provided by the current Medicaid program.
- No one—including friends or family—can take care of you for free. Long-term care must be purchased at market rates.
- If you choose Plan G, you can legally leave all your $100,000 as a bequest.
- Your bequest is not subject to any taxation under both Plan G and Plan H.

The comprehension questions discussed in the introduction are presented at this point (available on pages 57–59 of Survey 2). The respondents were then given the following screen and asked to record their decision.

Again, suppose you are 85 years old, live alone, rent your home, and pay all your own bills. You know with certainty that you will live only 12 more months, and that you will need help with ADLs for the entire 12 months. In this scenario, you have **$100,000**and must decide between either Plan G or Plan H.

- Plan G puts **all** of your $100,000 in an irrevocable bequest, and you will live in a publicly-funded nursing home.
- Plan H allows you to split your $100,000 between spending and bequests, and you are not eligible to live in a publicly funded nursing home.
  - From Plan H, you must designate whether to leave an irrevocable bequest, and if so, how much.
  - From Plan H funds you do not designate as a bequest, you will need to pay all your expenses, including long-term care and any other wants, needs, and discretionary purchases.

Given the $100,000 available to you, would you put money in Plan G and live in a publicly-funded nursing home, or would you instead put all $100,000 in Plan H and fund your own care?

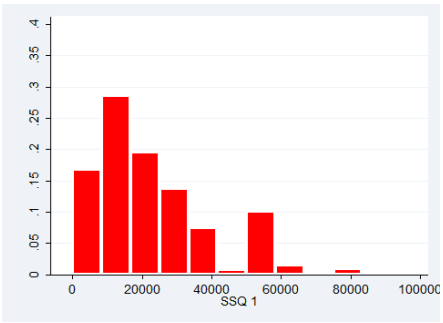
- Use Plan G that uses public care and leaves all money as a bequest.
- Use Plan H that divides my money between paying for my own care and a bequest.

Respondents were then asked the same question, exactly as presented in the above frame, at either the $20,000 or $1,000,000 level. Depending on their response, they were then asked to provide a wealth level at which they were indifferent using the slider.

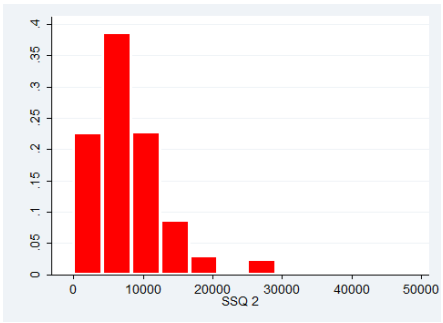
You have indicated that you would use Plan G (thus using public care and leaving all of your money as a bequest) if you had $20,000, but would use Plan H (thus funding both your own private LTC and bequest) if you had $100,000. What is the maximum amount of wealth for which you would still use Plan G (thus using public care and leaving all of your money as a bequest)?

5 SSQ Responses

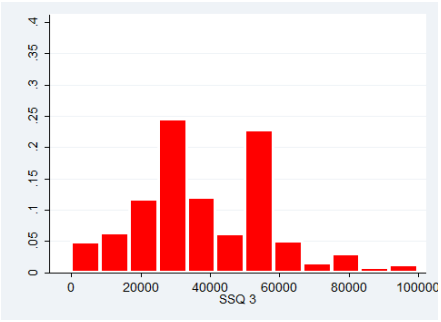
In Figure 1 we present the responses to all of the SSQ variants. In SSQs 1a and 1b, a response indicates how much income a respondent would be willing to risk. In SSQs 2a, 2b, 2c, 3a, 3b, a response indicates the amount of wealth allocated to the ADL state. In SSQ 4a a response indicates the wealth level at which a respondent is indifferent.



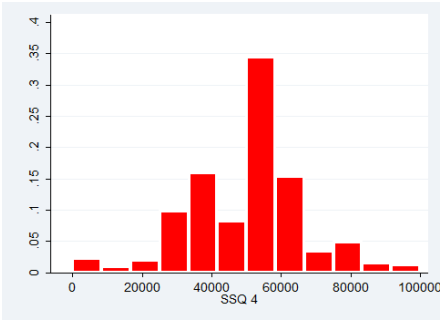
SSQ 1a



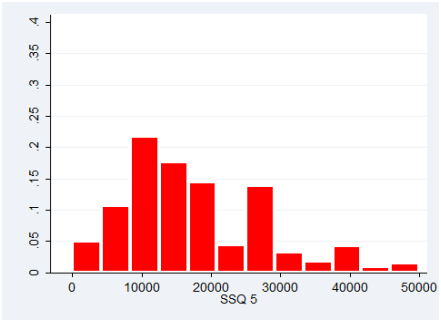
SSQ 1b



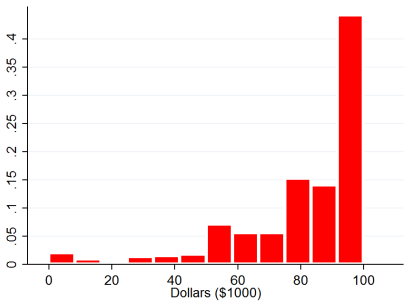
SSQ 2a



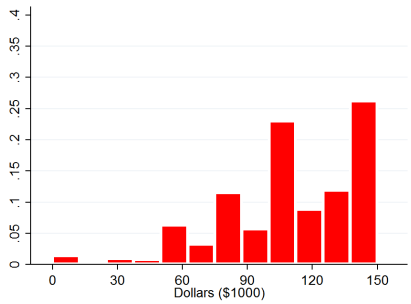
SSQ 2b



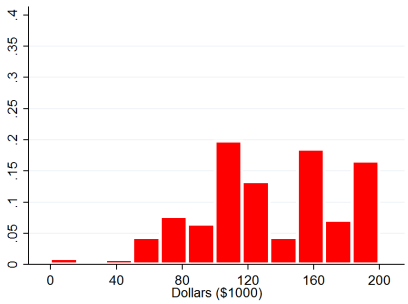
SSQ 2c



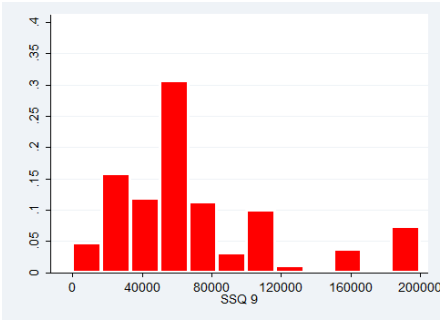
SSQ 3a



SSQ 3b



SSQ 3c



SSQ 4a

Figure 1: Distribution of Responses to SSQs

6 SSQ Optimal Responses

In this section we calculate the optimal responses to the SSQs consistent with given utility functional forms and values of preference parameters.

6.1 SSQ 1

SSQ 1 asks respondents to report the indifference point at which they are willing to risk doubling their next year's income against losing a significant amount of their lifetime wealth. Given that the probability of doubling income is given at $\pi = .5$ and the probability of losing a fraction of their income is given at $\pi = .5$, SSQ 1 iteratively asks for the values of λ for which

$$U(W) \leq \pi U(2W) + (1-\pi)U((1-\lambda)W).$$

The final response to SSQ 1 is designed to elicit $\lambda^\star$, at which the above holds with equality:

$$U(W) = \pi U(2W) + (1-\pi)U((1-\lambda^\star)W).$$

Specifying period utility to be given by a power function representing CRRA preferences,

$$U(W) = \frac{1}{1-\gamma}(W)^{1-\gamma},$$

yields

$$\begin{aligned}
\frac{1}{1-\gamma}(W)^{1-\gamma} &= \pi\frac{1}{1-\gamma}(2W)^{1-\gamma} + (1-\pi)\frac{1}{1-\gamma}((1-\lambda^\star)W)^{1-\gamma} \\
(W)^{1-\gamma} &= \pi(2W)^{1-\gamma} + (1-\pi)((1-\lambda^\star)W)^{1-\gamma} \\
(W)^{1-\gamma} - \pi(2W)^{1-\gamma} &= (1-\pi)((1-\lambda^\star)W)^{1-\gamma} \\
\frac{(W)^{1-\gamma} - \pi(2W)^{1-\gamma}}{(1-\pi)} &= ((1-\lambda^\star)W)^{1-\gamma} \\
\left(\frac{(W)^{1-\gamma} - \pi(2W)^{1-\gamma}}{(1-\pi)}\right)^{\frac{1}{1-\gamma}} &= W - W\lambda^\star \\
W\lambda^\star &= W - \left(\frac{(W)^{1-\gamma} - \pi(2W)^{1-\gamma}}{(1-\pi)}\right)^{\frac{1}{1-\gamma}}.
\end{aligned}$$

6.2 SSQ 2

SSQ 2 presents the respondent with a portfolio optimization problem with 2 Arrow securities. The first security pays off in the healthy state of the world. The second security pays off in the state of the world when long term care is needed.

The corresponding optimization problem is:

$$\begin{aligned}\max_{x_1,x_2} \quad & \pi\frac{x_1^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(\theta_{ADL})^{-\gamma}\,(x_2+\kappa_{ADL})^{1-\gamma}}{1-\gamma}\\ \text{s.t.} \quad & p_1x_1+p_2x_2\leq W\\ & x_2\geq 0.\end{aligned}$$

The variable x_2 is restricted to be positive to reflect the range of admissible responses in the VRI survey. This is only relevant if $\kappa_{ADL}>0$.

The Lagrangian is:

$$\mathcal{L}=\pi\frac{(\frac{W-p_2x_2}{p_1})^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(\theta_{ADL})^{-\gamma}\,(x_2+\kappa_{ADL})^{1-\gamma}}{1-\gamma}+\lambda(x_2).$$

The first order conditions are:

$$\begin{aligned}\pi\left(\frac{W-p_2x_2}{p_1}\right)^{-\gamma}\left(\frac{p_2}{p_1}\right)&=(1-\pi)\,(\theta_{ADL})^{-\gamma}\,(x_2+\kappa_{ADL})^{-\gamma}+\lambda\\ 0&=\lambda x_2.\end{aligned}$$

If $x_2>0$, then $\lambda=0$, and

$$\begin{aligned}(W-p_2x_2)\left(\frac{\pi p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}&=p_1\theta_{ADL}(x_2+\kappa_{ADL})\\ W\left(\frac{\pi_1p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}-p_1\theta_{ADL}\kappa_{ADL}&=p_1\theta_{ADL}x_2+p_2x_2\left(\frac{\pi p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}\\ x_2&=\frac{W\left(\frac{\pi p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}-p_1\theta_{ADL}\kappa_{ADL}}{p_1\theta_{ADL}+p_2\left(\frac{\pi p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}}.\end{aligned}$$

If $x_2=0$, then

$$\begin{aligned}\pi_1\left(\frac{W}{p_1}\right)^{-\gamma}\left(\frac{p_2}{p_1}\right)&=(1-\pi_1)(\theta_{ADL}\kappa_{ADL})^{-\gamma}+\lambda\\ \lambda&=\pi_1\left(\frac{W}{p_1}\right)^{-\gamma}\left(\frac{p_2}{p_1}\right)-(1-\pi_1)(\theta_{ADL}\kappa_{ADL})^{-\gamma}>0.\end{aligned}$$

Thus, the optimal policy rule is

$$x_2=\begin{cases}0 & \text{if } \pi\left(\frac{W}{p_1}\right)^{-\gamma}\left(\frac{p_2}{p_1}\right)-(1-\pi)(\theta_{ADL}\kappa_{ADL})^{-\gamma}>0\\ \frac{W\left(\frac{\pi p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}-p_1\theta_{ADL}\kappa_{ADL}}{p_1\theta_{ADL}+p_2\left(\frac{\pi p_2}{p_1(1-\pi)}\right)^{-\frac{1}{\gamma}}} & \text{otherwise.}\end{cases}$$

6.3 SSQ 3

In SSQ 3 the agent solves a portfolio maximization problem with assets that represent "locked boxes." There is no stochastic state and the agent knows for certain that he will need LTC and then leave a bequest. The first security is available for his spending while alive and the second security pays out as a bequest at death.

The corresponding optimization problem is:

$$\begin{aligned} \max_{x_1,x_2} \quad & (\theta_{ADL})^{-\gamma} \ \frac{(x_1+\kappa_{ADL})^{1-\gamma}}{1-\gamma} + (\theta_{beq})^{-\gamma} \ \frac{(x_2+\kappa_{beq})^{1-\gamma}}{1-\gamma} \\ \text{s.t.} \quad & x_1 + x_2 \leq W \\ & x_1 \geq 0; \ x_2 \geq 0. \end{aligned}$$

The variables x_1 and x_2 are restricted to be positive to reflect the range of admissible responses in the VRI survey. This is only relevant if $\kappa_{ADL} > 0$ or $\kappa_{beq} > 0$.

Substituting in the budget constraint and writing out the Lagrangian yields:

$$\mathcal{L} = (\theta_{ADL})^{-\gamma} \ \frac{(x_1+\kappa_{ADL})^{1-\gamma}}{1-\gamma} + (\theta_{beq})^{-\gamma} \ \frac{(W-x_1+\kappa_{beq})^{1-\gamma}}{1-\gamma} + \lambda_1 x_1 + \lambda_2 (W - x_1)$$

The first order conditions and associated complimentary slackness conditions are as follows

$$\begin{aligned} (\theta_{ADL})^{-\gamma} (x_1+\kappa_{ADL})^{-\gamma} - (\theta_{beq})^{-\gamma} (W - x_1 + \kappa_{beq})^{-\gamma} + \lambda_1 - \lambda_2 &= 0 \\ \lambda_1 x_1 &= 0 \\ \lambda_2 (W - x_1) &= 0 \end{aligned}$$

Note that at most one of the constraints can bind at any given time.
If $W > x_1 > 0$, then

$$\begin{aligned} (\theta_{ADL})^{-\gamma} (x_1+\kappa_{ADL})^{-\gamma} &= (\theta_{beq})^{-\gamma} (W - x_1 + \kappa_{beq})^{-\gamma} \\ (\theta_{ADL} + \theta_{beq}) x_1 &= \theta_{beq}(W + \kappa_{beq}) - \theta_{ADL}\kappa_{ADL} \\ x_1 &= \frac{\theta_{beq}(W+\kappa_{beq}) - \theta_{ADL}\kappa_{ADL}}{\theta_{ADL} + \theta_{beq}}. \end{aligned}$$

If $x_1 = W$ then

$$\lambda_2 = (\theta_{ADL})^{-\gamma} (W + \kappa_{ADL})^{-\gamma} - (\theta_{beq}\kappa_{beq})^{-\gamma} \geq 0$$

If $x_1 = 0$ then

$$\lambda_1 = (\theta_{beq})^{-\gamma} (W + \kappa_{beq})^{-\gamma} - (\theta_{ADL}\kappa_{ADL})^{-\gamma} \geq 0.$$

Thus, the optimal policy rule is

$$x_1 = \begin{cases} 0 & \text{if } \left(\theta_{beq}\right)^{-\gamma}\left(W+\kappa_{beq}\right)^{-\gamma} - \left(\theta_{ADL}\kappa_{ADL}\right)^{-\gamma} > 0 \\ W & \text{if } \left(\theta_{ADL}\right)^{-\gamma}\left(W+\kappa_{ADL}\right)^{-\gamma} - \left(\theta_{beq}\kappa_{beq}\right)^{-\gamma} > 0 \\ \frac{\theta_{beq}(W+\kappa_{beq})-\theta_{ADL}\kappa_{ADL}}{\theta_{ADL}+\theta_{beq}} & \text{otherwise.} \end{cases}$$

6.4 SSQ 4

SSQ 4 asks for the level of wealth at which an agent is indifferent between paying for private LTC or utilizing Medicaid provided government long term care. An agent is given an option of paying for private LTC or utilizing public care and leaving his entire wealth as a bequest. That is, SSQ 4 attempts to elicit the W^* that satisfies

$$U_2(\psi_G) + \beta v(W^*) = U_2(x_1) + \beta v(W^* - x_1).$$

SSQ 4 uses the x_1 allocations derived in SSQ 3. First, note that the agent will never choose to consume $x_1 < \psi_G$. Thus, the optimal x_1 from SSQ 3 is

$$x_1 = \begin{cases} W & \text{if } \left(\theta_{ADL}\right)^{-\gamma}\left(W+\kappa_{ADL}\right)^{-\gamma} - \left(\theta_{beq}\kappa_{beq}\right)^{-\gamma} > 0 \\ \frac{\theta_{beq}(W+\kappa_{beq})-\theta_{ADL}\kappa_{ADL}}{\theta_{ADL}+\theta_{beq}} & \text{otherwise.} \end{cases}$$

To compute the indifference wealth level W^*, we first show that there exists a W^* that satisfies the indifference condition. First, in the case where $W = 0$, clearly the LHS is larger. In the case with $W > 0$, from SSQ 3, note that x_1 and x_2 are increasing in W and for sufficiently large W, $x_1 < W$.

No suppose that for all $W > 0$ the LHS is at least as large as the RHS. This would imply

$$U_2(\psi_G) - U_2(x_{i,1}) \geq \beta v(x_{i,2}) - \beta v(W).$$

Since x_1 is increasing in W, the LHS of the above inequality must be strictly negative for large enough W. In contrast, since x_2 is increasing in W and v is strictly increasing and strictly less than zero, the RHS of the above inequality converges to zero as W grows. This would imply, for sufficiently large W,

$$0 > \lim_{W\to\infty} U_2(\psi_G) - U_2(x_{i,1}) > \lim_{W\to\infty} \beta v(x_{i,2}) - \beta v(W) = 0,$$

which is a contradiction.

The indifference level of wealth W^* can be shown to be unique following similar arguments, using the monotonicity and continuity of the utility functions.

If $x_1 < W$, then the indifferent point W^* satisfies

$$\begin{aligned} U_2(\psi_G) + \beta v(W^*) &= U_2(x_1) + \beta v(W^* - x_1) \\ U_2(\psi_G) + \beta v(W^*) &= U_2\left(\frac{\theta_{beq}(W+\kappa_{beq}) - \theta_{ADL}\kappa_{ADL}}{\theta_{ADL}+\theta_{beq}}\right) + \beta v\left(W^* - \frac{\theta_{beq}(W+\kappa_{beq}) - \theta_{ADL}\kappa_{ADL}}{\theta_{ADL}+\theta_{beq}}\right). \end{aligned}$$

There does not exist an analytical expression for the $W^\star$ which solves this expression, but it is easily computed numerically.

If $x_1 = W$, then the indifferent point W^* satisfies

$$U_2(\psi_G) + \beta v(W^*) \quad = \quad U_2(W^*) + \beta v(0),$$

which can be solved numerically. Thus, we can express the solution to this problem as

$$W^\star \text{ solves } \begin{cases} U_2(\psi_G) + \beta v(W^*) = U_2(W^*) + \beta v(0) & \text{if } \theta_{ADL}\left(\alpha W^* + \kappa_{ADL}\right)^{-\gamma}\alpha - \beta\theta_{beq}(\kappa_{beq})^{-\gamma} > 0 \\ U_2(\psi_G) + \beta v(W^*) = U_2\left(\frac{\theta_{beq}(W+\kappa_{beq})-\theta_{ADL}\kappa_{ADL}}{\theta_{ADL}+\theta_{beq}}\right) & \\ \qquad\qquad + \beta v\left(W^* - \frac{\theta_{beq}(W+\kappa_{beq})-\theta_{ADL}\kappa_{ADL}}{\theta_{ADL}+\theta_{beq}}\right) & \text{otherwise.} \end{cases}$$

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